



Fault Fundamentals

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Power Plant Protection Track

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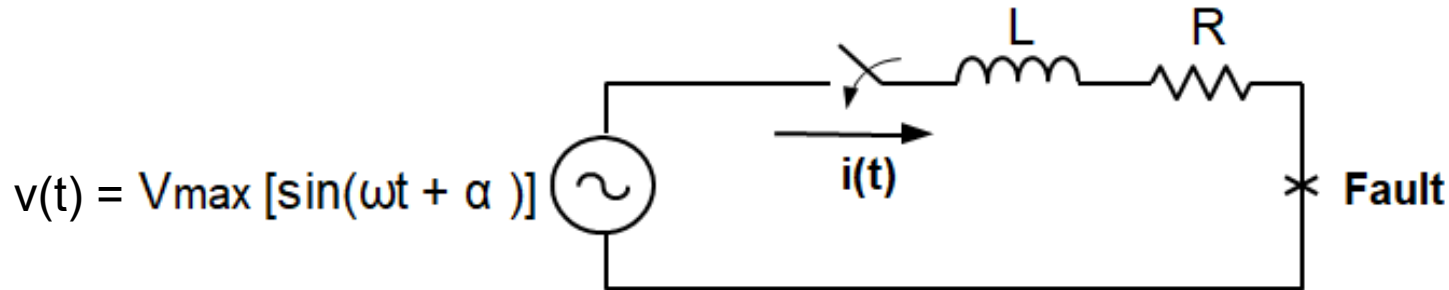
Fault Calculations – Topics

- **Short Circuit Current**
- **Per Unit Math**
- **Symmetrical Components**
- **Fault Types**
- **Calculation Examples**

Short Circuit Current

What is a fault?

- suddenly applied Short Circuit



KVL:
$$v(t) = L \frac{di}{dt} + Ri$$

$$V_{\max} [\sin(\omega t + \alpha)] = L \frac{di}{dt} + Ri$$

where:

- $v(t)$ = instantaneous value of the voltage at t seconds
- V_{\max} = amplitude or max, peak voltage of the sinusoidal waveform
- ω = angular frequency in radians ($\omega = 2\pi f$ and f is the frequency in Hz)
- α = Fault Inception Angle (FIA) i.e. the point in the sinusoidal waveform where the fault is applied or when breaker closes into fault

Short Circuit Current

$$V_{\max}[\sin(\omega t + \alpha)] = L \frac{di}{dt} + Ri$$

Solve this equation for current with respect to time:

$$i(t) = \frac{V_{\max}}{Z} \left[\sin(\omega t + \alpha - \Theta) - \sin(\alpha - \Theta) e^{-\frac{Rt}{L}} \right]$$

This can be represented as such:

$$\mathbf{i(t) = i_{ac}(t) + i_{dc}(t)}$$

- R is the resistance
- L is the inductance
- $X = 2\pi f L$
- X is the reactance
- $Z = R + jX$
- Z is the impedance
- Θ is the angle of Z

Therefore, fault current can be represented as (2) separate components:

$$\mathbf{i_{ac}(t) = \frac{V_{\max}}{Z} \sin(\omega t + \alpha - \Theta)}$$

Steady state current

$$\mathbf{i_{dc}(t) = \frac{V_{\max}}{Z} \sin(\alpha - \Theta) e^{-\frac{Rt}{L}}}$$

Transient current or DC offset

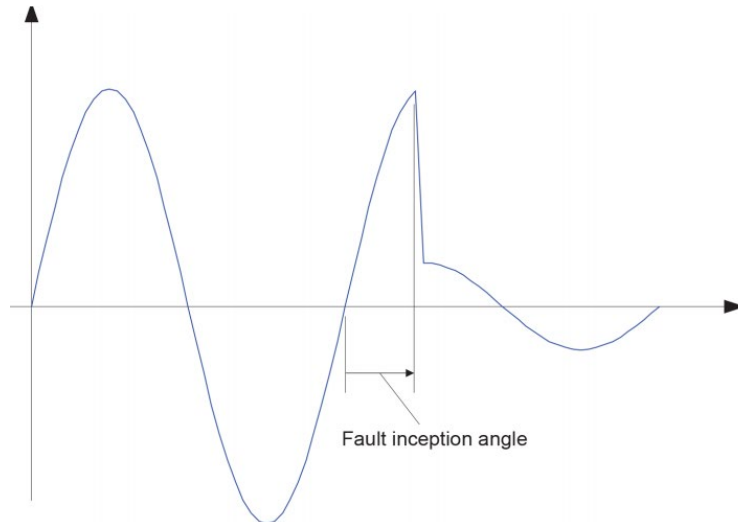
First, let's discuss the DC portion:

What is the Fault Inception Angle (FIA = α)?

- FIA is the point or angle on voltage waveform when fault occurs in reference to the nearest preceding zero crossing.
- DC offset current is referred to as just “DC offset” as current is implied.
- FIA dictates the initial magnitude of DC offset.

$$i(t) = \frac{Vm}{Z} \left[\sin(\omega t + \alpha - \Theta) - \sin(\alpha - \Theta) e^{-\frac{Rt}{L}} \right]$$

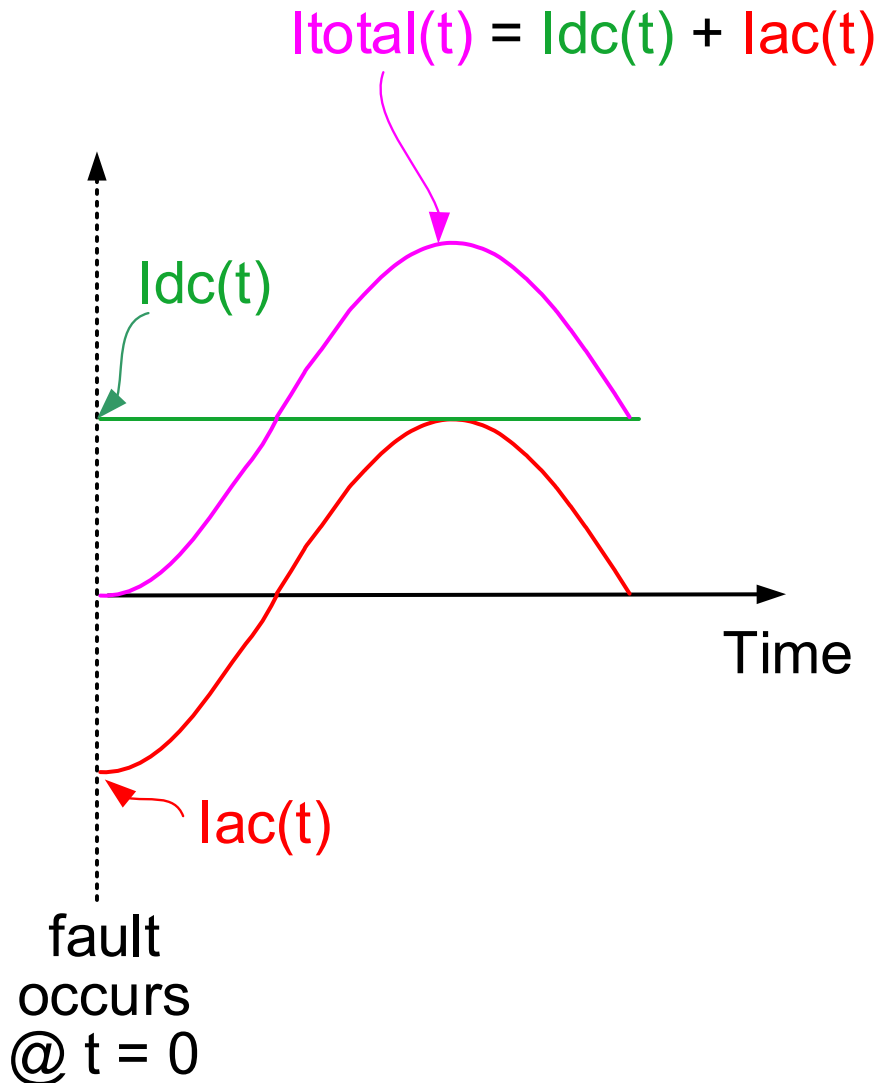
with $R = 0, e^0 = 1,$ therefore $i(t) = \frac{Vm}{Z} \left[\sin(\omega t + \alpha - \Theta) - \sin(\alpha - \Theta) \right]$



DC Offset

- Why is “Offset” added to “DC” for the DC portion of total fault current?
 - ✓ When a fault occurs, it shifts the sine wave asymmetrically with respect to the time axis i.e. creates ”**offset**”.
- This offset is necessary to maintain some basic principles of electricity at the precise moment that the current makes a sudden change:
 1. (**ELI** the ICE man) Inductive circuit, current lags voltage by 90° .
 - Therefore, if a fault occurs at a voltage zero crossing ($V=0$), then the current will be at a positive or negative maximum.
 2. Current through an inductor cannot change instantaneously, $I(0^-)=I(0^+)$.
- Assume gen carrying no load prior to fault (e.g. breaker syncs closed into a fault):
 1. If fault occurs at $V=0$, then $I=\max$.
 2. Because at no load, $I(0^-)=0$ prior to fault, but because current cannot change instantaneously, therefore $I(0^+)=0$ just after the fault as well.
 - ✓ How can $I=\max$ and $I=0$ at the same time?
- **DC offset to the rescue – it allows compliance with both principles simultaneously**

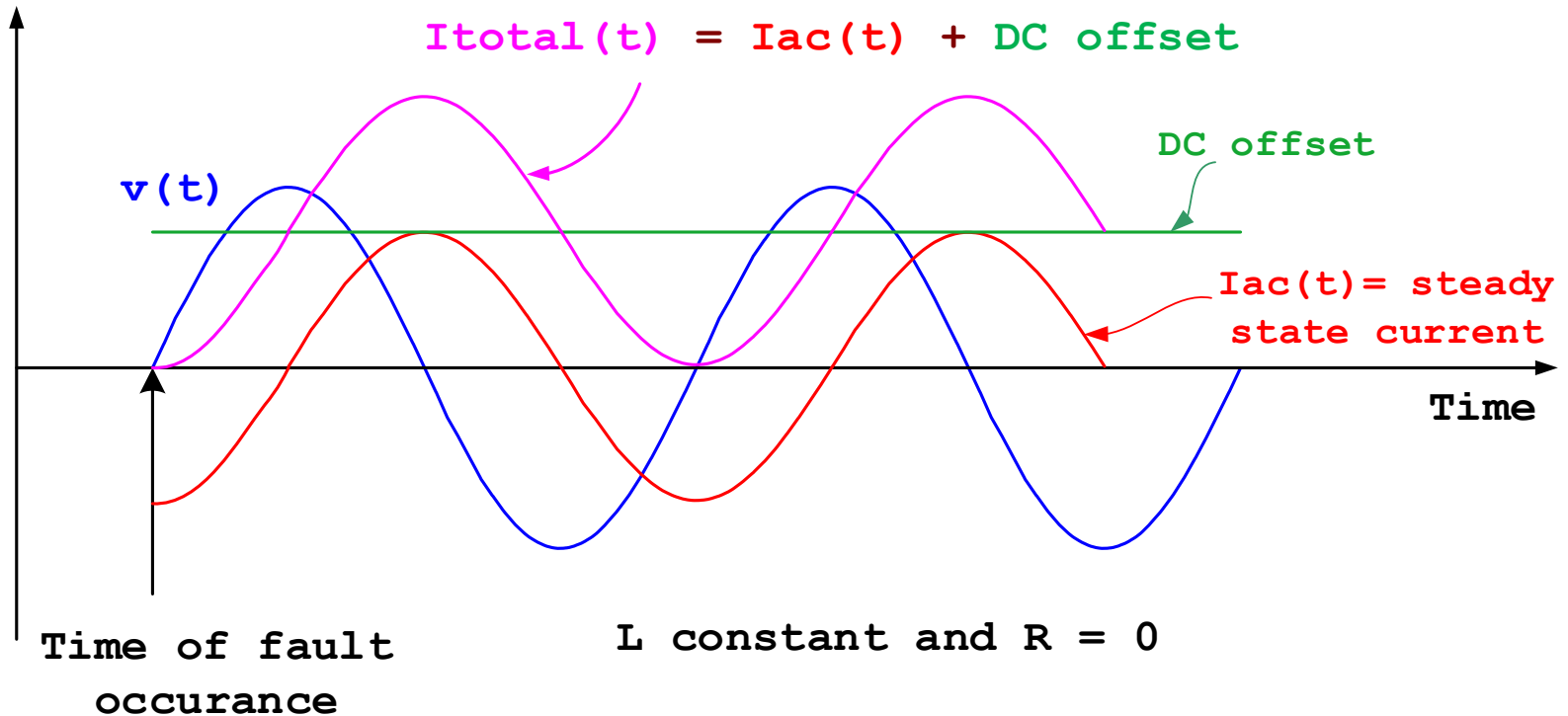
DC Offset



- DC offset current is created at the instant a fault occurs.
- At $t=0$, DC offset is equal to AC component but opposite polarity, $I_{dc} = - I_{ac}$.
- Therefore, the total fault current = 0 at $t=0$, $I_{ac} + I_{dc} = 0$ at $t=0$.
- At $t=0$, $I_{total}(t) = I_{ac}(t) + I_{dc} = 0$

DC Offset

- Fault at positive-going voltage zero crossing, results in $I_{AC} = -\text{max}$, therefore $I_{DC} = +\text{max}$



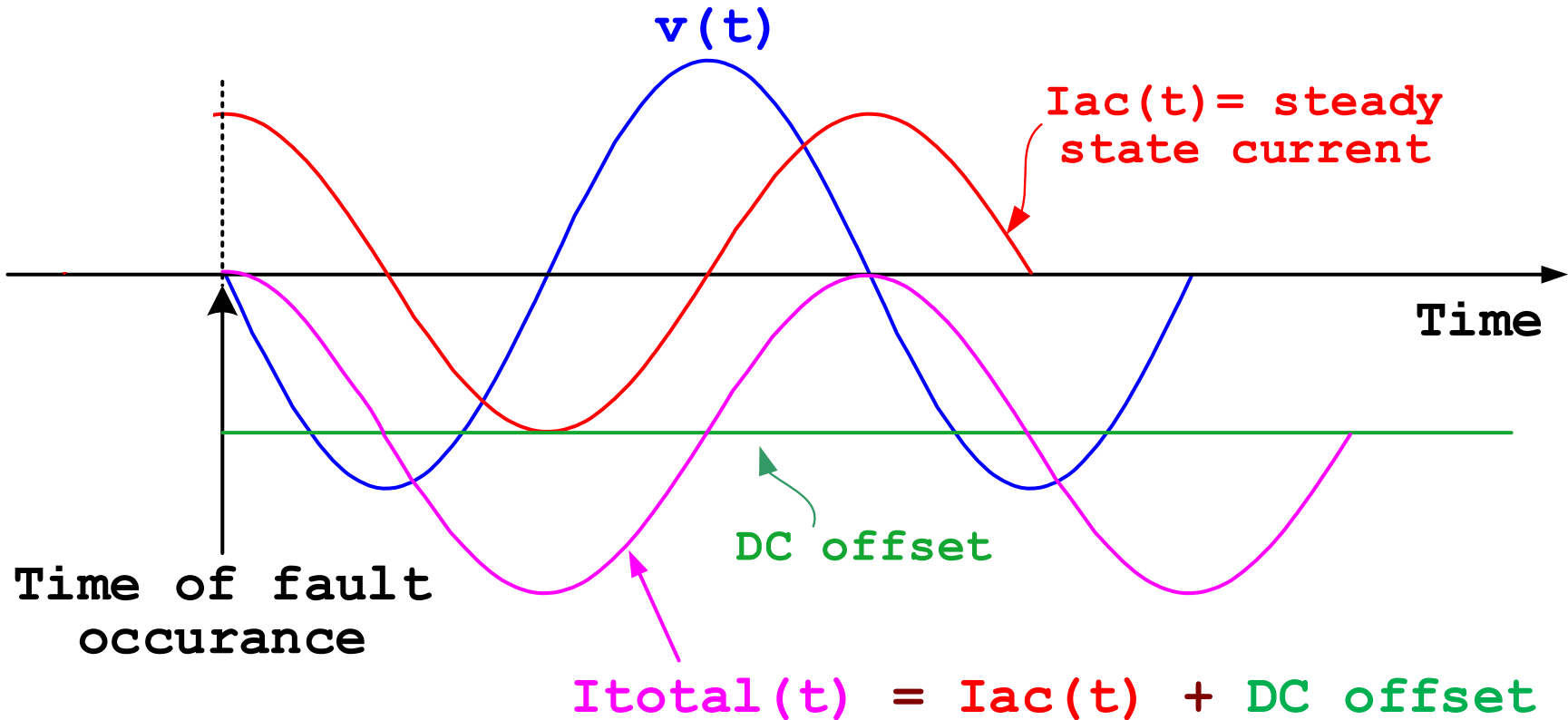
$$i(t) = i_{ac}(t) + i_{dc}(t) = \frac{V_{max}}{Z} \left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta) e^{-\frac{Rt}{L}} \right], \theta \text{ is the angle of } Z$$

$$i_{dc}(t) = -\frac{V_{max}}{Z} \sin(\alpha - \theta) e^{-\frac{Rt}{L}} = -\frac{V_{max}}{Z} \sin(0 - 90^\circ) e^{-0} = -\frac{V_{max}}{Z} * -1 * 1 = \frac{V_{max}}{Z}$$

$$i_{ac}(t) = \frac{V_{max}}{Z} \sin(\omega t + 0 - 90^\circ)$$

DC Offset

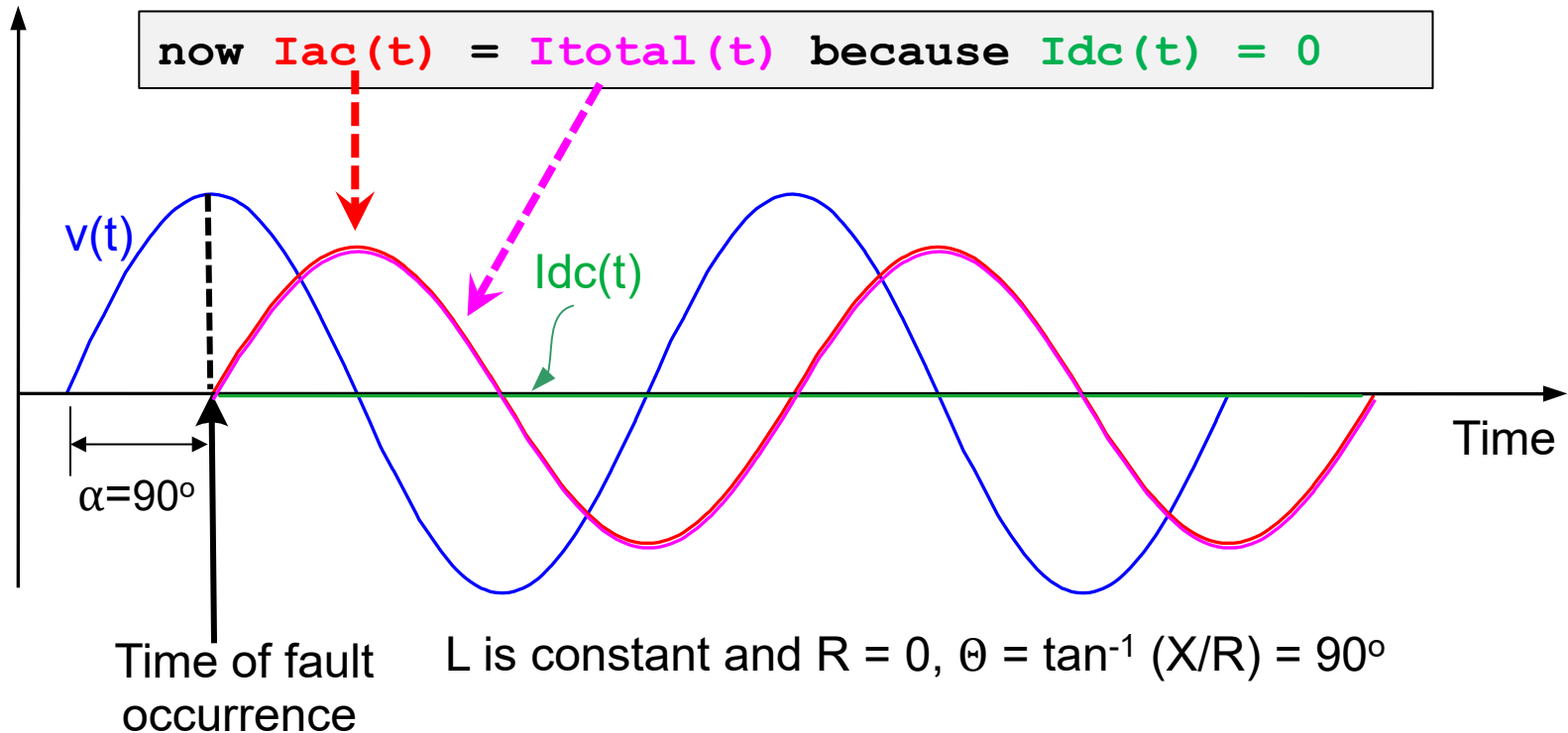
- Fault at negative-going voltage zero crossing, results in $I_{AC} = + \text{max}$, therefore $I_{DC} = - \text{max}$



DC Offset

Fault at max voltage results in min DC offset current

- fault @ V_{\max} , then per ELI, $i(t)$ is at a zero crossing so DC Offset = 0 A.



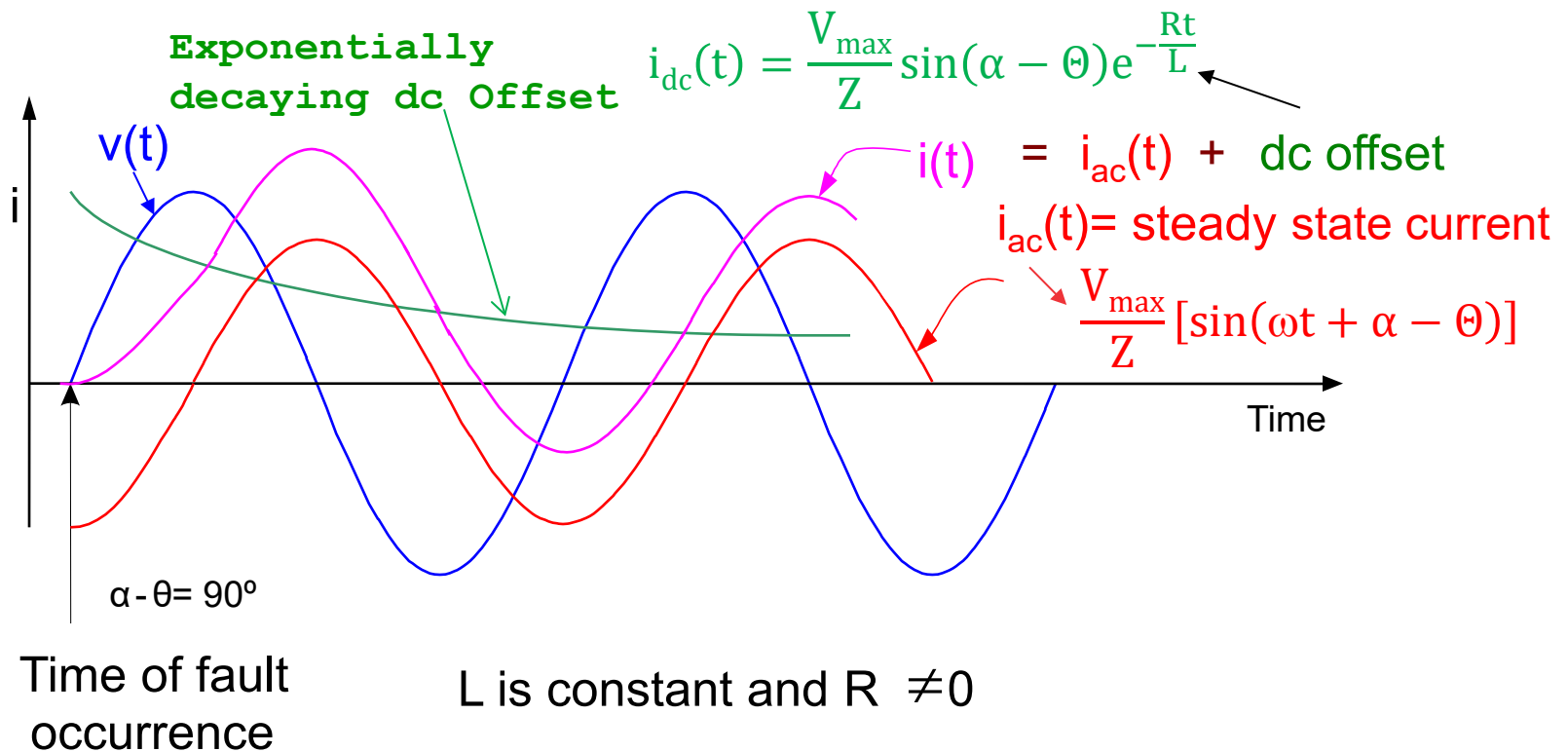
$$i_{dc}(t) = \frac{V_{\max}}{Z} \sin(\alpha - \theta) e^{-\frac{Rt}{L}} = \frac{V_{\max}}{Z} \sin(90^\circ - 90^\circ) e^{-0} = \frac{V_{\max}}{Z} \sin(0^\circ) e^{-0} = 0 \text{ A}$$

$$i(t) = i_{ac}(t) + i_{dc}(t) = \frac{V_{\max}}{Z} [\sin(\omega t + \alpha - \theta) - 0] = \frac{V_{\max}}{Z} [\sin(\omega t)]$$

DC Offset

Fault at Voltage Zero when $R \neq 0$

- The higher the X/R ratio, the longer it takes for dc offset to decay to zero.
- The smaller the R, the higher the X/R and the higher the time constant τ .



Why do we care about DC Offset?

1) Breaker sizing

- Slow breakers – may not be as critical to calculate the amount of DC offset that is possible because the DC offset component may decay down to zero by the time a “slow” breaker operates.
- Fast breakers – may be more important to calculate the maximum amount of DC offset that is possible as it may still be present when breaker opens.

2) Instantaneous overcurrent relay (or 50DT relays with very short time delays)

- May be important to calculate the maximum possible DC offset for older E/M and some Static relays that do not filter out the DC component.
- Modern digital relays use filtering e.g. DFT filtering that filter most of the DC; therefore, it may not be as necessary to calculate the max asymmetrical fault current.

3) CT Saturation Calculations (CT dimensioning, CT modelling, etc)

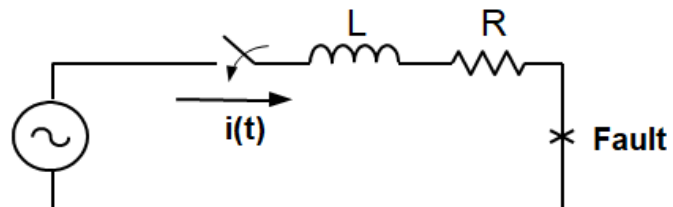
4) Event Analysis

5) POW (Point-On-Wave) switching

1) DC Offset – Breaker sizing

- System planner says a 2-cycle breaker is required for maintaining system stability
- How big a breaker do we need to specify in terms of fault interrupting capability?
- Offset will vary depending at what point on the waveform the fault occurs
- Size breaker for worst case conditions – max possible DC offset, V zero crossing

- From the simple RL circuit:



- Assume $V_{rms} = 2.4 \text{ KV}$, $R = 0.1 \ \Omega$, $X_L = 2 \ \Omega$:

- $$I_{ac}(RMS) = \frac{V}{R+jXL} = \frac{2400}{0.1+j2} = 1199 \angle -87^\circ \text{ A} \approx 1200 \text{ A}$$

- Multiply this RMS value by $\sqrt{2}$ to get the peak value, which will be the max possible DC offset:

- $$I_{peak} = I_{dcmax} = \sqrt{2} * I_{ac}(RMS) = \sqrt{2} * 1200 = 1697 \approx 1700 \text{ A}$$

1) DC Offset – Breaker sizing (continued)

- The DC offset will exponentially decay with time constant τ , based on system X/R from the source to the fault location (the higher the X/R, the slower the decay):

- $$\tau = \frac{L}{R} = \frac{X}{\omega R} = \frac{2}{377 * 0.1} = 0.053 \text{ seconds}$$

- The transient DC offset will be:

- $$Idc(t) = Idcmax * e^{-\frac{t}{\tau}} = \sqrt{2} * Iac(RMS) * e^{-\frac{t}{\tau}}$$

- At the 2-cycle breaker interrupt time ($2/60 = 0.033$ sec):

- $$Idc@2cyc = \sqrt{2} * 1200 * e^{-\frac{0.033}{0.053}} = 905 \text{ A}$$

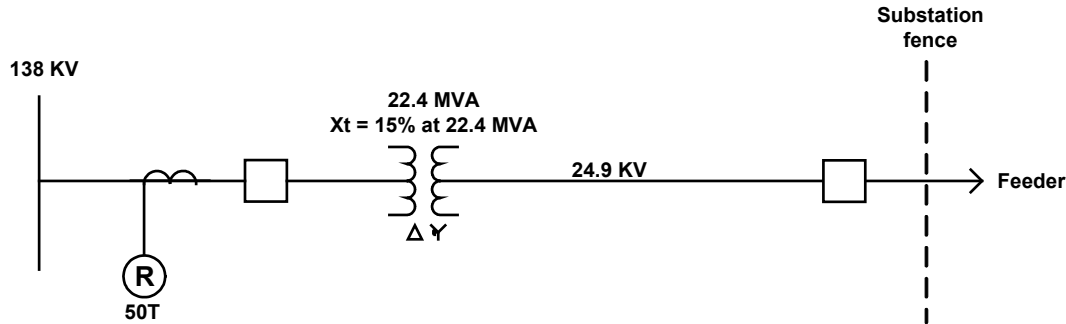
- The total transient RMS current, accounting for both the AC and DC components:

- $$I_{total} = \sqrt{Iac(RMS)^2 + Idc@2cyc^2} = \sqrt{1200^2 + 905^2} = 1503 \text{ A} \quad (2 \text{ cycle breaker})$$

$$= 1207 \text{ A} \quad (\text{for 8 cycle breaker})$$

so DC portion is almost fully gone

2) DC Offset – Instantaneous Overcurrent



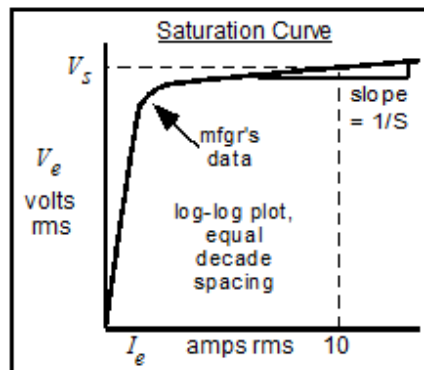
Set > max asymmetrical fault current for LS fault as seen by HS relay (E/M relays):

- *inrush; asym LS fault* < 50T Pickup < *min phase fault on HS*
- $I_{asymm} = 1.6 * I_{F SYM}$ **(1.6 is rule of thumb)**
- Max DC offset is $\sqrt{2} * I_{ac}$
- $$I(t) = \sqrt{I_{ac}^2 + I_{dc}(t)^2} = \sqrt{I_{ac}^2 + (\sqrt{2} * I_{ac} * e^{-\frac{t}{\tau}})^2} = I_{ac} \sqrt{1 + 2e^{-\frac{2t}{\tau}}}$$
- At $t = 0$, max total fault current is $\sqrt{3} * I_{ac} = 1.732 * I_{ac}$
- **Because there are no 0 cycle breakers, a general “rule of thumb” is to use the 1.6 factor, instead of the theoretical max $\sqrt{3}$ factor.**
- **Instantaneous Overcurrent relays are not truly “instantaneous”, and they may also have a time delay setting making them a Definite Time relay, so depending on what the time delay is set at, the DC offset may not need to be considered e.g. < 30 cycles or so (to be very conservative), then may disregard DC.**

3) DC Offset – CT Saturation

INPUT PARAMETERS:

	ENTER:		
Inverse of sat. curve slope =	S =	22	---
RMS voltage at 10A exc. current =	Vs =	200	volts rms
Turns ratio = $n2/1=$	N =	40	---
Winding resistance =	Rw =	0.300	ohms
Burden resistance =	Rb =	0.500	ohms
Burden reactance =	Xb =	0.500	ohms
System X/R ratio =	XoverR =	12.0	---
Per unit offset in primary current =	Off =	0.50	-1<Off<1
Per unit remanence (based on Vs) =	λ_{rem}	0.50	---
Symmetrical primary fault current =	Ip =	2,000	amps rms

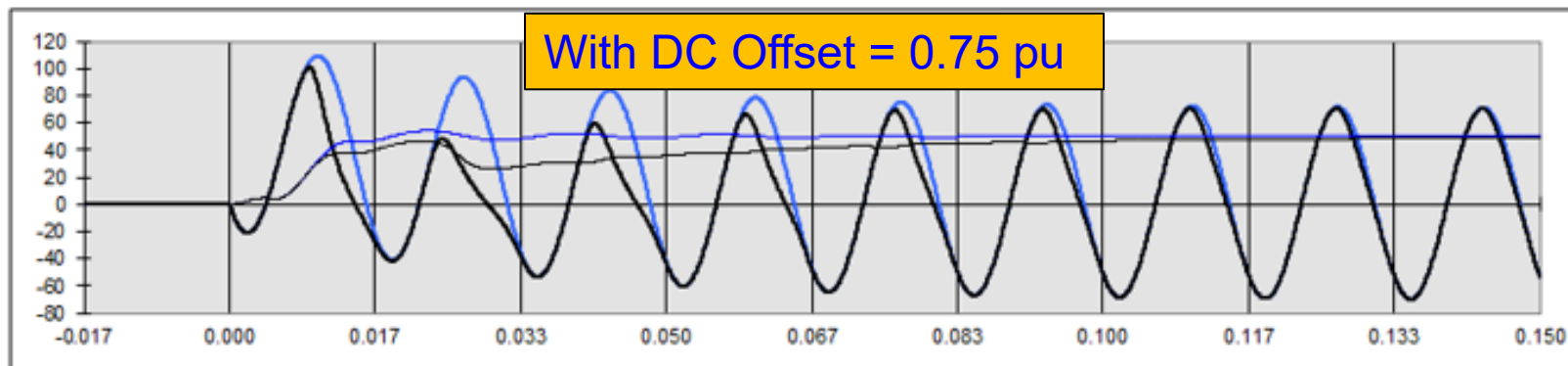
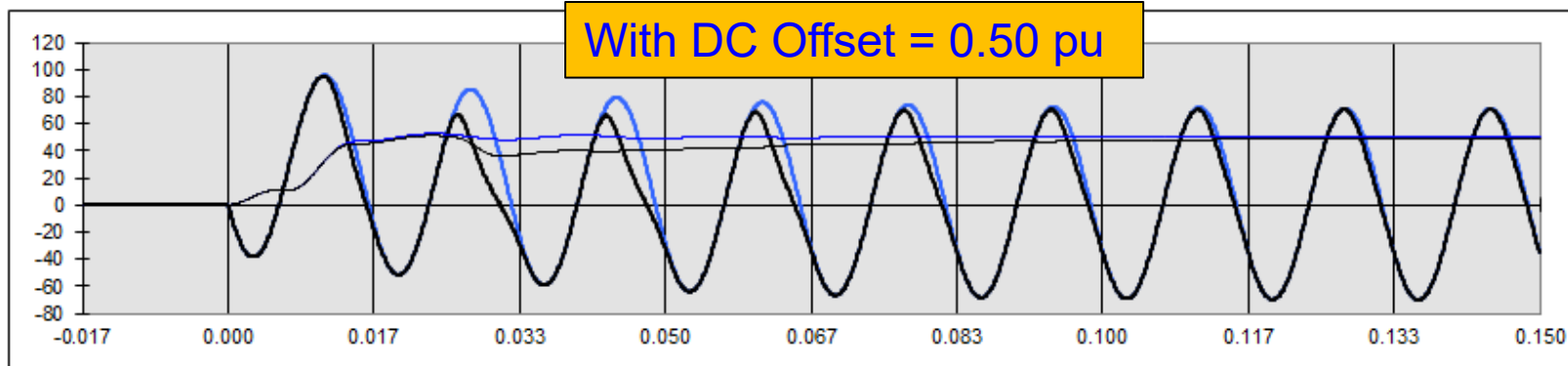


CALCULATED:

Rt = Total burden resistance = $R_w + R_b =$	0.800	ohms
pf = Total burden power factor =	0.848	---
Zb = Total burden impedance =	0.943	ohms
Tau1 = System time constant =	0.032	seconds
Lamsat = Peak flux-linkages corresponding to Vs =	0.750	Wb-turns
ω = Radian freq =	376.99	rad/s
RP = Rms-to-peak ratio =	0.34584	---
A = Coefficient in instantaneous ie versus lambda curve: $i_e = A * I^S$:	1.61E+04	---
dt = Time step =	0.000083	seconds
Lb = Burden inductance =	0.00133	henries

Thick lines: **Ideal (blue)** and **actual (black)** secondary current in amps vs time in seconds.

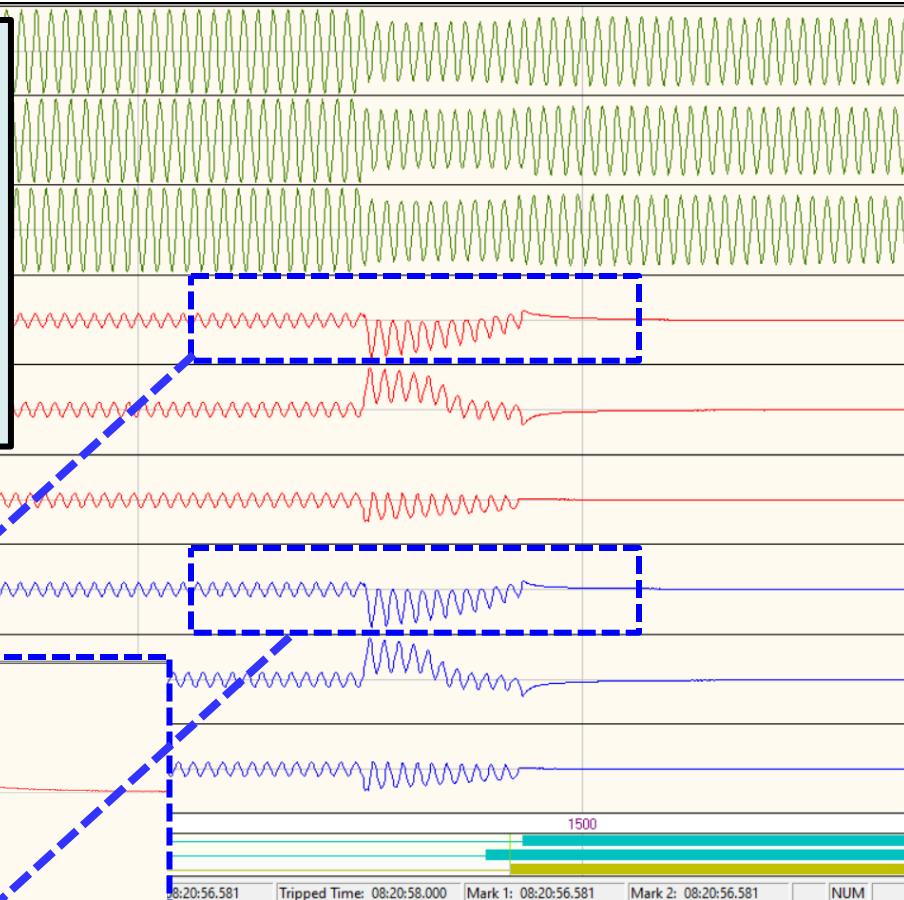
Thin lines: **Ideal (blue)** and **actual (black)** secondary current extracted fundamental rms value, using a simple DFT with a one-cycle window.



4) DC Offset – Event Analysis

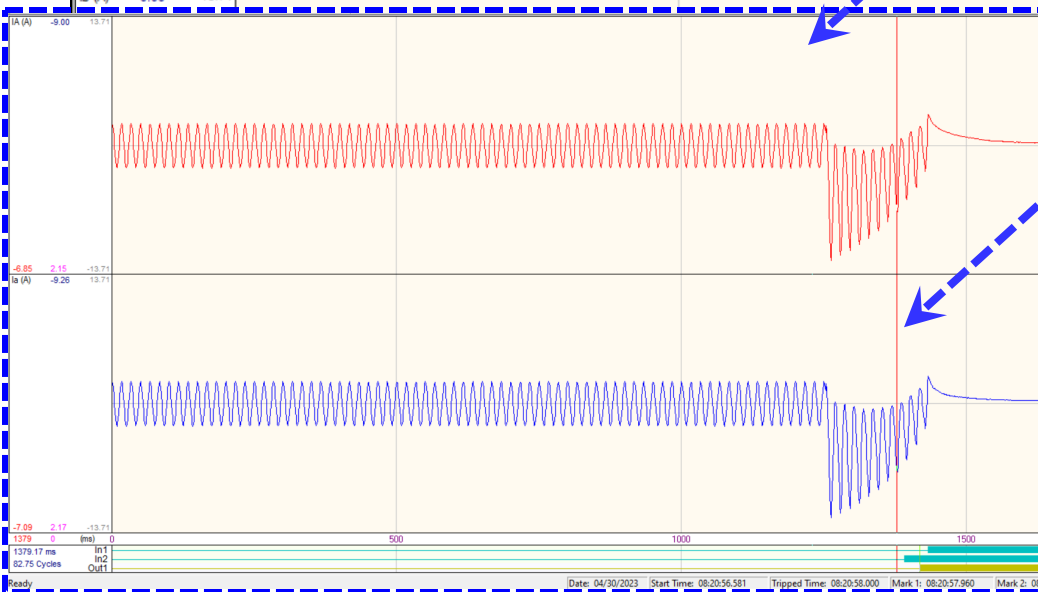
Severe DC offset can result in missing or delayed zero crossings which can damage breaker and other equipment, endanger personnel, and cause system instability.

Different relay protection elements may look for a zero crossing to process the measured current and detect the event, resulting in delayed tripping if there are delayed zero crossings.



Most breaker's are designed to interrupt the fault current at a zero crossing. If breaker interrupts current with no zero crossings, it may not be able to extinguish the arc resulting in extensive damage.

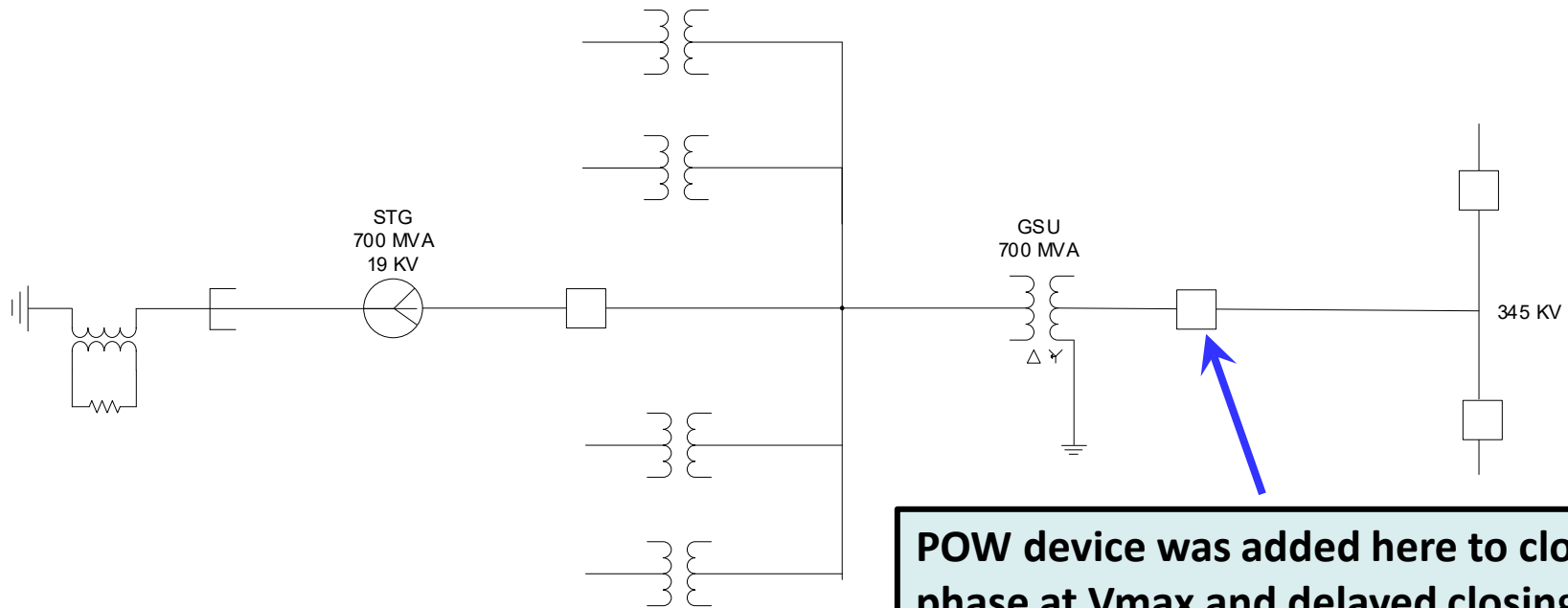
Gen breaker should be rated to interrupt current with DC offset including with delayed zero crossings.



5) POW (Point-On-Wave) switching

Consider the transformer inrush current when closing in this GSU HS breaker which energizes the xfmr pack (GSU + 4 UATs) during startup or to back energize the plant load during outage:

- Transient mitigation strategies such as pre-insertion resistors or surge arrestors, only dampen the transient after it has already occurred.
- POW (Point-On-Wave) switching can prevent DC offset current before it occurs i.e. to energize xfmr pack, close breaker at V_{max} to give min DC offset i.e. min inrush current.
- However, must also consider possible residual flux in the GSU and UATs.



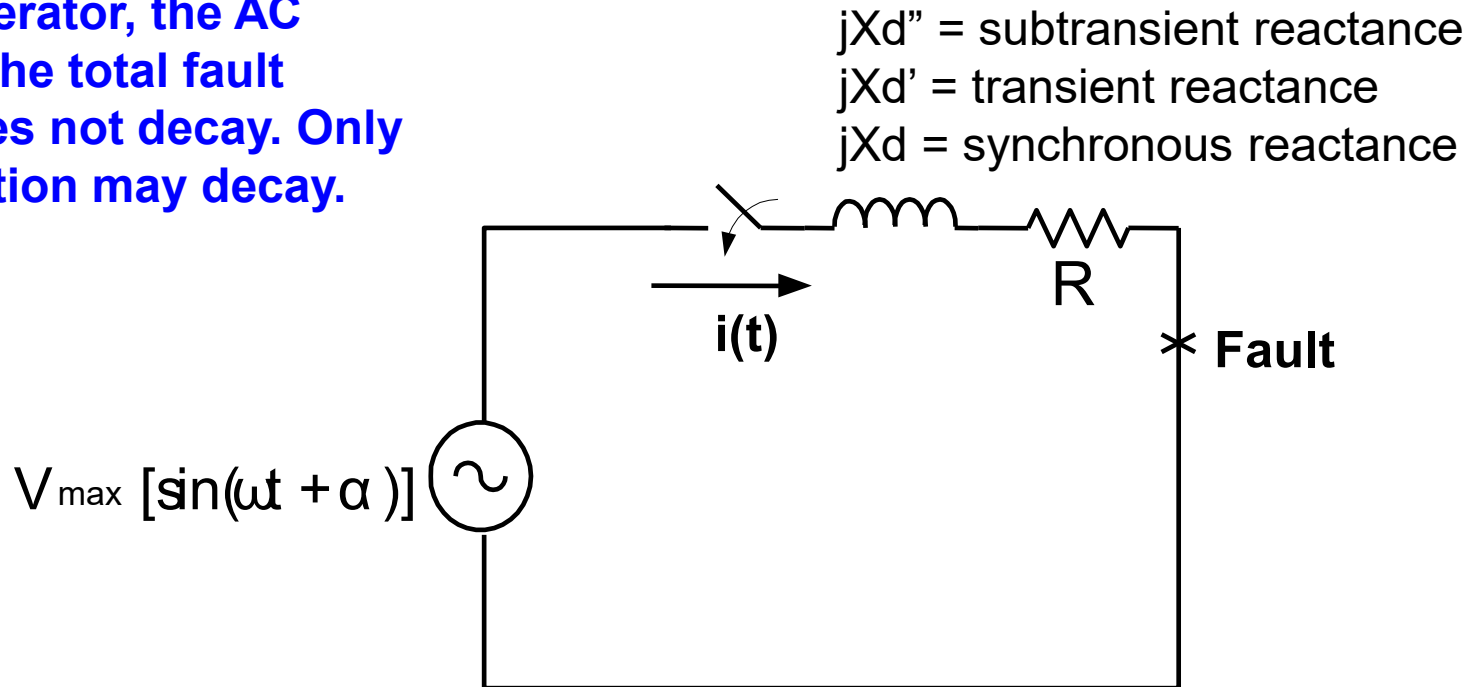
POW device was added here to close A-phase at V_{max} and delayed closing of B & C phases by 7 cycles to automatically demagnetize the residual flux in the cores of the transformer pack.

Next, discuss the AC portion of fault current:

What is the generator decrement curve?

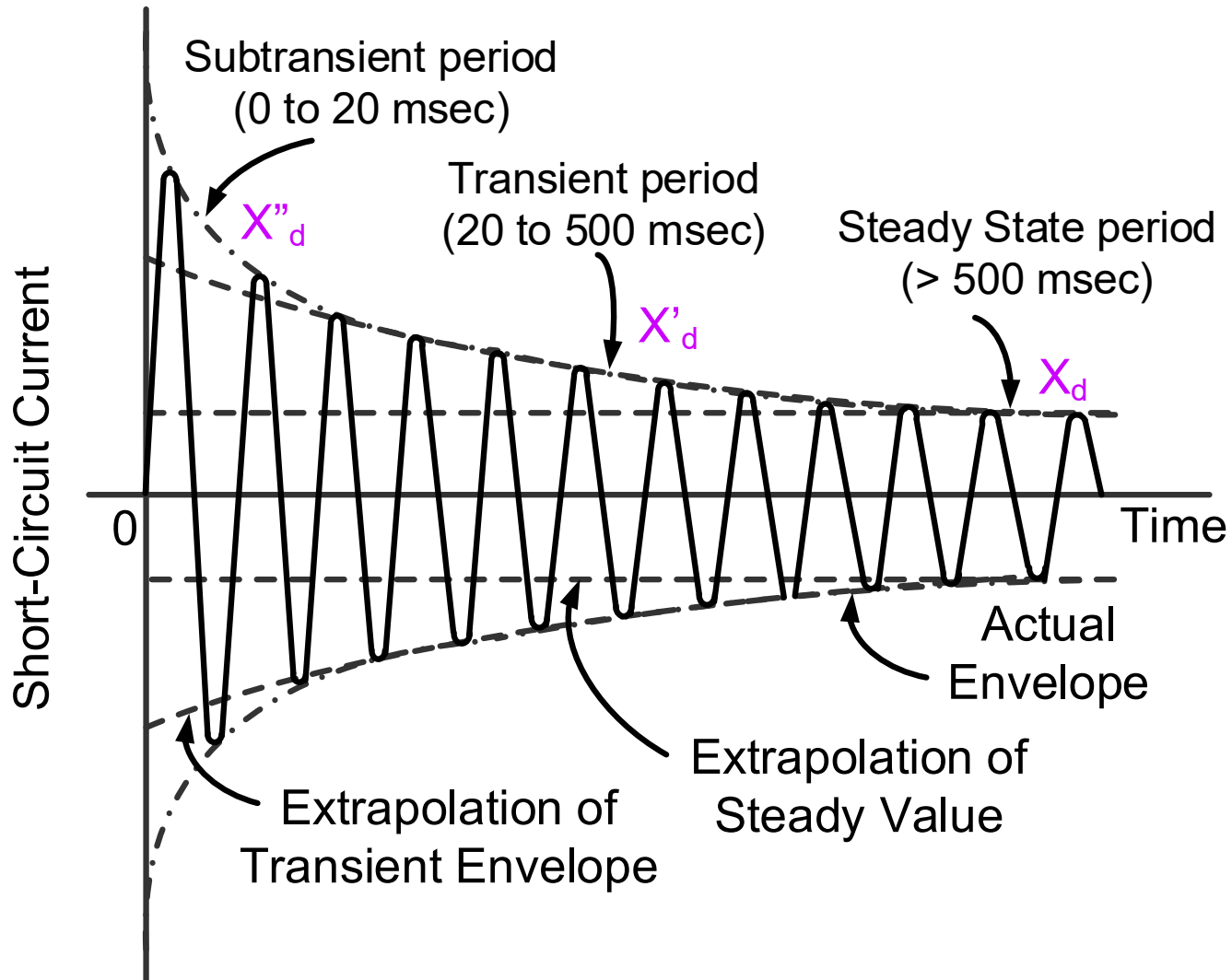
- This is the amount of fault current that a generator can contribute to a fault with respect to time.
- It is the time-decaying AC component of the total fault current for faults near generators due to the varying generator reactance.

NOTE: For faults far away from a generator, the AC portion of the total fault current does not decay. Only the DC portion may decay.



AC portion of the total fault current

Generator Short-Circuit AC Current Decay



AC portion of the total fault current

Generator Decrement Curve Equation:

$$I_{ac}(t) = (I_{d''} - I_{d'}) * e^{\frac{-t}{T_{d''}}} + (I_{d'} - I_d) * e^{\frac{-t}{T_{d'}}} + I_d$$

- Represents the fault current decay after the generator breaker opens, but prior to the field breaker opening assuming no field forcing.

Example calculation of the AC portion of the fault current that a generator can contribute at 0.50 seconds to a 3Φ fault:

$$I_{ac}(t) = I_{pu} * I_{base}$$

$$I_{ac}(t) = \left[(I_{d''} - I_{d'}) * e^{-\frac{t}{T_{d''}}} + (I_{d'} - I_d) * e^{-\frac{t}{T_{d'}}} + I_d \right] * I_{base}$$

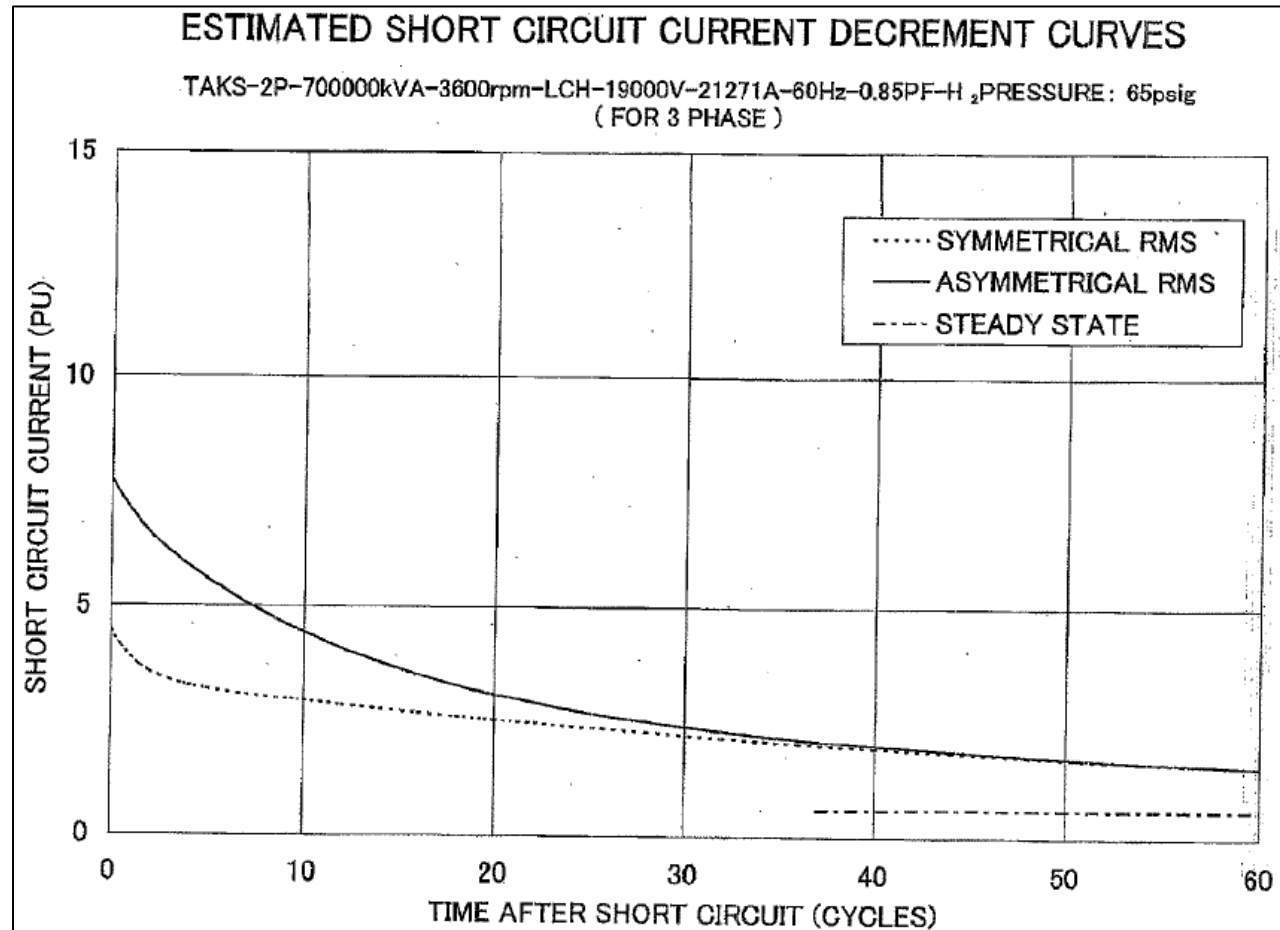
$$I_{ac}(t) = \left[\left(\frac{1}{X_{d''}} - \frac{1}{X_{d'}} \right) * e^{-\frac{t}{T_{d''}}} + \left(\frac{1}{X_{d'}} - \frac{1}{X_d} \right) * e^{-\frac{t}{T_{d'}}} + \frac{1}{X_d} \right] * \frac{S_{gen}}{\sqrt{3} * V_{LL}}$$

$$I_{ac}(t) = \left[\left(\frac{1}{0.224} - \frac{1}{0.295} \right) * e^{-\frac{0.50}{0.025}} + \left(\frac{1}{0.295} - \frac{1}{1.66} \right) * e^{-\frac{0.50}{1.4}} + \frac{1}{1.66} \right] * \frac{700 M}{\sqrt{3} * 19 K}$$

$$I_{ac}(t) = 54,298 \text{ pri amps}$$

AC portion of the total fault current

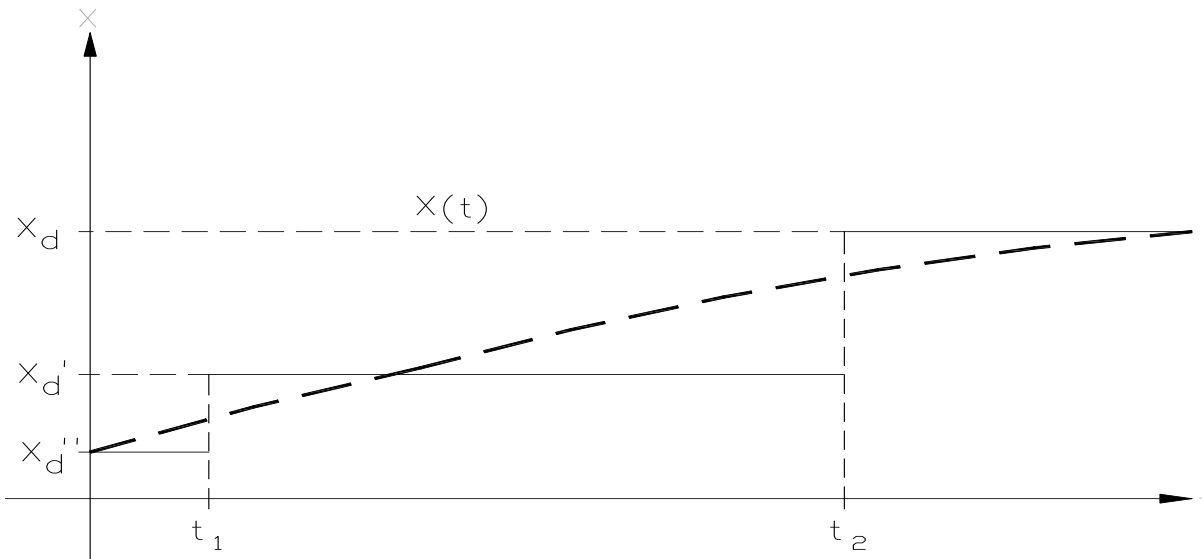
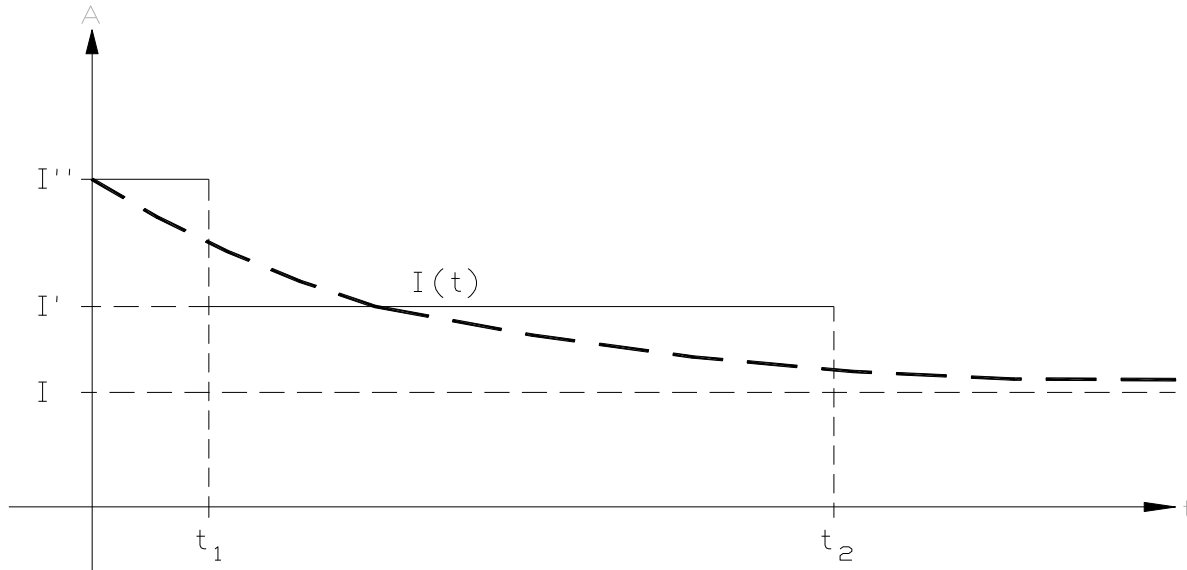
- Mfgs typically provide the generator decrement curves with time on the horizontal axis:



- Whereas when plotting the gen decrement curve against your 51V curve, then time is typically on the vertical axis.

AC portion of the total fault current

$I(t)$ and $X(t)$ are inversely proportional:



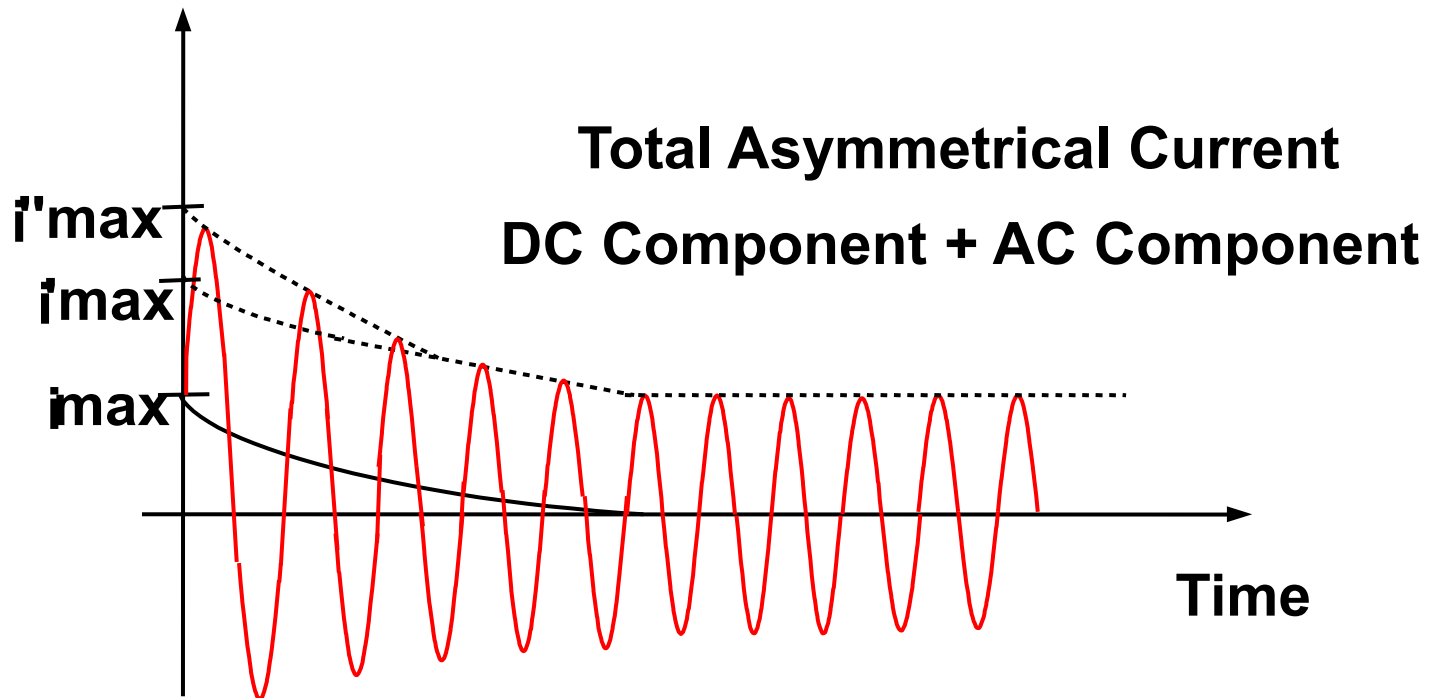
AC portion of the total fault current

When to use which generator reactance: X_d'' , X_d' , X_d ?

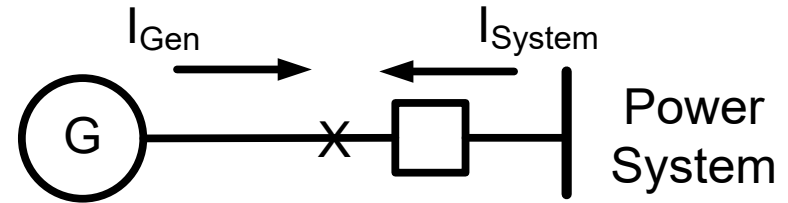
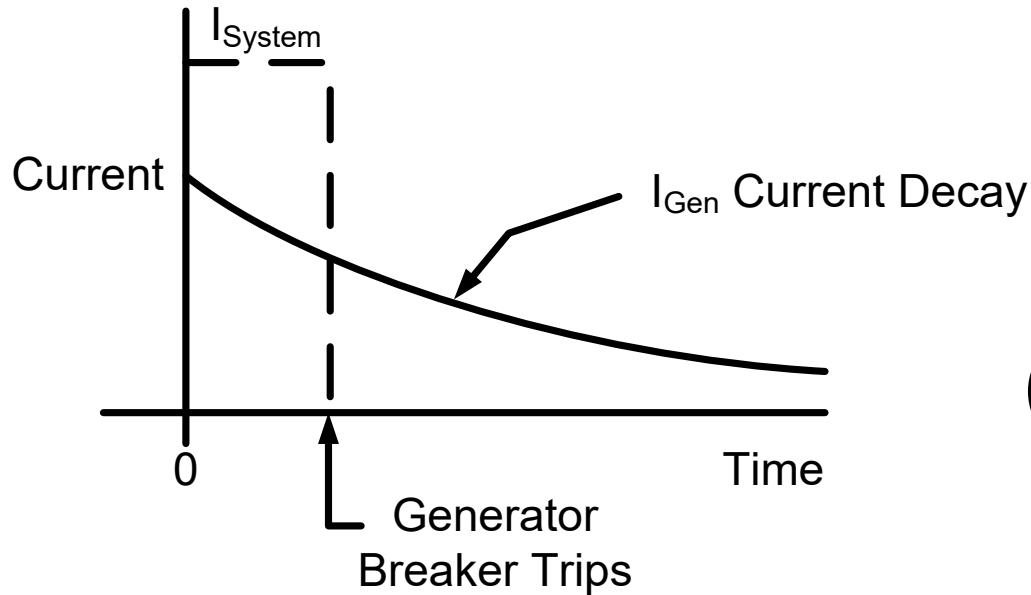
- Instantaneous relay pickup (e.g. 50) criteria may use fault current calculated from just the X_d'' subtransient reactance.
- Use the generator decrement curve equation to calculate the fault current at a certain time if you want to account for the relay's operate/pickup time.
- Or if it is a definite time relay then may calculate the fault current at the relay's time delay setting plus the operate/pickup time.
- Transient stability studies may use the X_d' transient reactance although some transient stability study software may use all 3 reactances with the generator time constants.
- Fault studies may use X_d'' or X_d' or some fault study software may use all 3 reactances with the generator time constants.

Total fault current: $i(t) = i_{ac}(t) + i_{dc}(t)$

- **Asymmetrical Fault Current** – Asymmetry with respect to the time axis indicates there is some DC offset current present.
- **Unsymmetrical or Unbalanced Fault Current** – Current from unbalanced faults ($\Phi\Phi$, ΦG , $\Phi\Phi G$). NOTE: Some resources may interchange the terms asymmetrical, unsymmetrical, and unbalanced. I prefer not to use the term “Unsymmetrical” at all, and instead just say “Unbalanced” faults or if I am referring to the symmetry of the current waveform relative to the time axis, I use the term “Asymmetrical”.



Total Fault Current

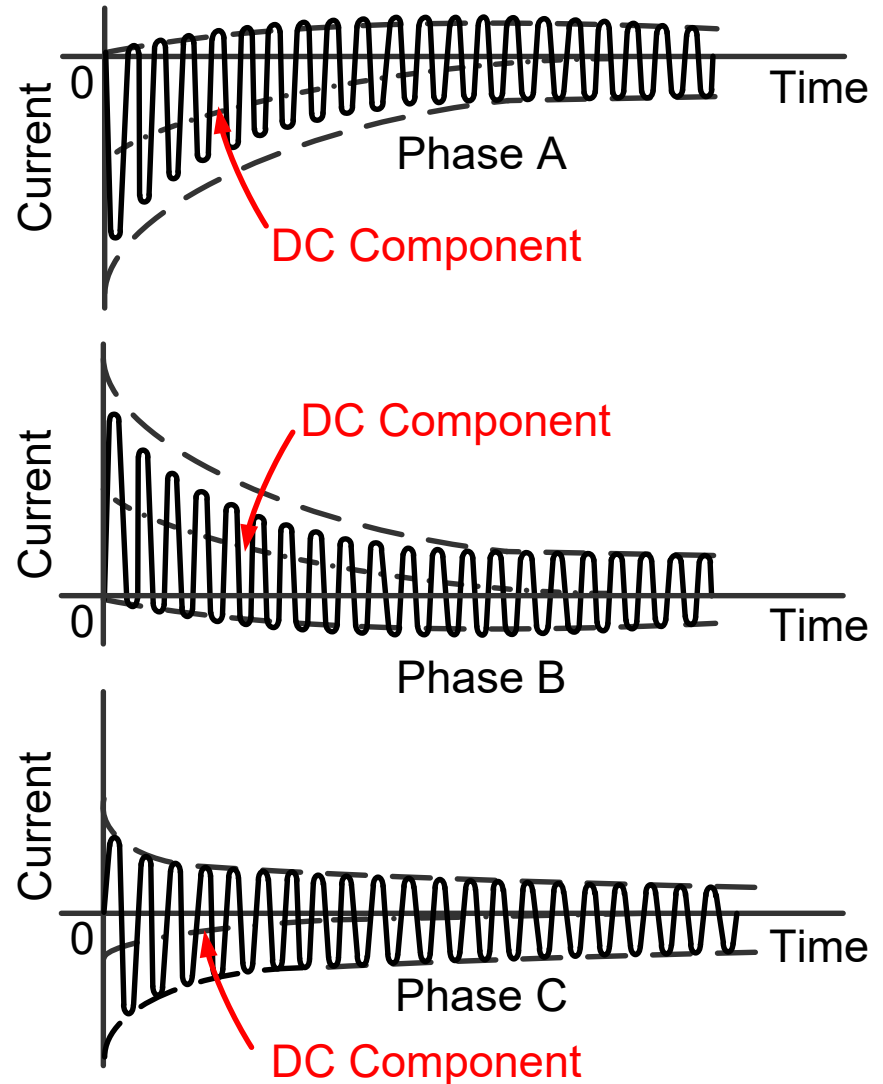


- After the gen breaker trips, the system contribution to the fault is cleared immediately.
- The generator contribution will take some time to decay based on the generator's time constants due to trapped magnetic flux.
- If field forcing is applied, the decay will be slower.
- Then, when the field breaker is tripped, the excitation current to the rotor is cut off causing field flux to collapse, further accelerating current decay.

Total Fault Current

3 Φ gen fault current, total = AC + DC

Effect of DC Offset
shifts are different
on each phase

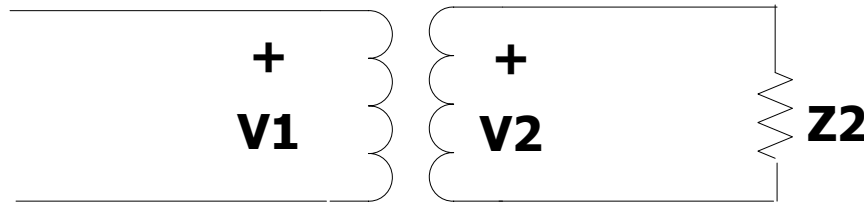


Per Unit Math

Short Circuit Calculations

- Calculate the RMS symmetrical short circuit current:
- $I = \frac{V}{Z}$ (ac portion of the fault current)
- Where V is the RMS driving voltage
- Z is the thevenin equivalent system impedance from the fault point back to and including the source
- max asymmetrical (ac + dc) fault current $\approx 1.6 * \text{symmetrical fault current}$
- When doing fault calculations using actual values, must transfer impedances thru transformers:

$$Z1 = Z2 * \frac{V1^2}{V2^2}$$



- This can get a bit tedious. **Per Unit math to the rescue.**

Per Unit Math

- Parameters fall into relatively narrow range, so can more easily identify incorrect values
 - Convert all system parameters to a common base
 - All components at all voltage levels are combined
 - Transformers become “transparent” to calculations regardless of winding configuration
 - Operating system current and voltage values can then be derived
 - Can use pu or % but pu has advantage in that $\text{pu} * \text{pu} = \text{pu}$; whereas $\% * \%$ must be divided by 100 to get %
- ✓ **Always use pu for calculations by hand, rather than %**

Per Unit Math

Establish two base quantities:

- $S_{3\Phi}$ – **Base Power** – 3 phase
 - V_{LL} – **Base Voltage** – line to line
-
- Other quantities are derived with basic power equations

Per Unit Math

$$I_{base} = \frac{S_{base}}{\sqrt{3} * V_{LLbase}}$$

$$Z_{base} = \frac{V_{LLbase}^2}{S_{base}}$$

Per Unit Math

$$\text{per unit} = \frac{\text{actual}}{\text{base}}$$

$$V_{pu} = \frac{V_{actual}}{V_{base}}$$

$$I_{pu} = \frac{I_{actual}}{I_{base}}$$

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

Per Unit Math

$$Z_{pu2} = Z_{pu1} * \frac{V_{base1}^2}{V_{base2}^2} * \frac{S_{base2}}{S_{base1}}$$

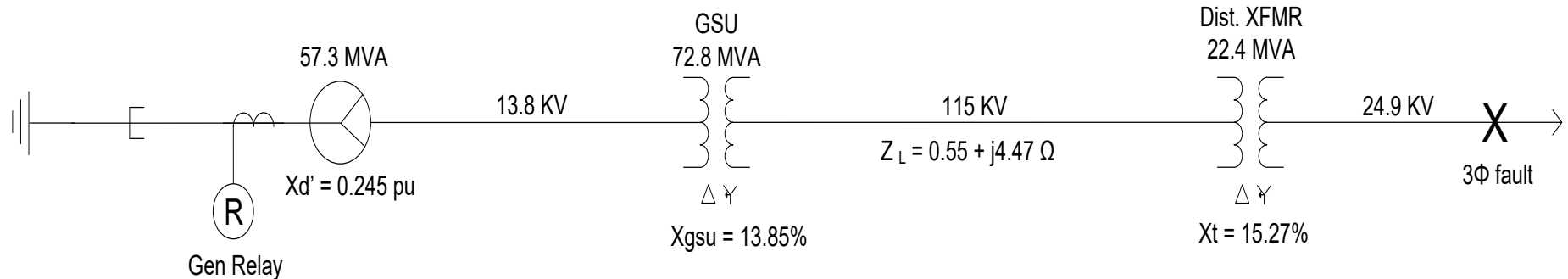
- Only use at same voltage level where slightly different voltage bases exist.
- **NOTE: Do not use this equation to transfer impedance from 1 side of xfmr to the other.**

$$Z_{pu2} = Z_{pu1} * \frac{S_{base2}}{S_{base1}}$$

- Use if equipment voltage ratings are the same as system base voltages.

Per Unit Math

e.g. using per unit math to easily calculate fault current thru transformers:

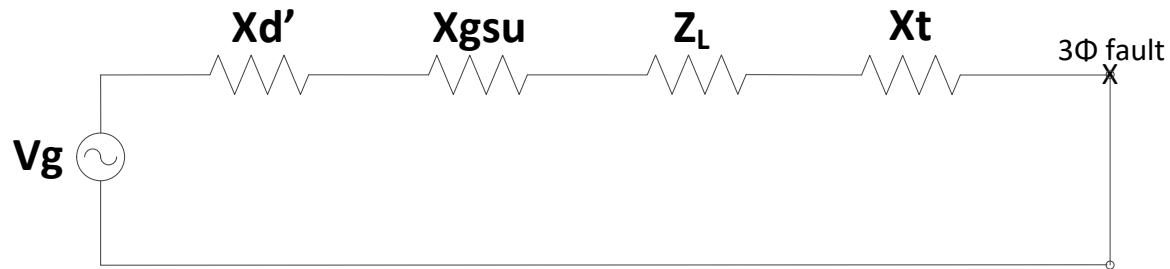


- What will Gen Relay see for a 3Φ fault on the 24.9 KV Distribution Line?
- Get all impedances on same base then can add directly.
- Define $S_{base} = 57.3$ MVA
 - **NOTE:** For gen relays, use gen max rating for S_{base} , whereas for transmission protection fault calcs, typically use 100 MVA as base

Per Unit Math

- $Xd' = 0.245 \text{ pu}$
- $X_{gsu} = X_{old} * \left(\frac{S_{base_new}}{S_{base_old}} \right) = 0.1385 * \left(\frac{57.3}{72.8} \right) = 0.109 \text{ pu}$
- $Z_L = \frac{Z_{actual}}{Z_{base}} = \frac{Z_L}{\frac{V_{LL}^2}{S_{base}}} = \frac{(0.55 + j4.47)}{\frac{115^2}{57.3}} = 0.0024 + j0.0194 \text{ pu}$
- $X_t = X_{old} * \left(\frac{S_{base_new}}{S_{base_old}} \right) = 0.1527 * \left(\frac{57.3}{22.4} \right) = 0.391 \text{ pu}$

Per Unit Math



- $I_{pu} = \frac{V_{pu}}{Z_{pu}} = \frac{1}{X_{d'} + X_{gsu} + Z_L + X_t} = \frac{1}{j0.245 + j0.109 + 0.0024 + j0.0194 + j0.391} = 1.3 pu$
- $I_{actual} = I_{pu} * I_{base} = 1.3 * \frac{57.3 * 1000}{\sqrt{3} * 13.8} = 3116 pri\ amps$

Symmetrical Components

“Provides a practical methodology for understanding and analyzing power system operation during unbalance conditions”

“In a sense Symmetrical Components can be called the language of the relay engineer or technician”

Protective Relaying - Principles and Applications
J. Lewis Blackburn

Symmetrical Components

- Standard power system equations assume a balanced 3Φ system.
- What to do when it is not balanced?
- Need a technique to de-construct the unbalanced currents into a set of balanced currents to do the calculations.
- **Symmetrical Components to the rescue.**
- On 6/28/1918, CL Fortescue of Westinghouse presented a Symmetrical Components paper at an AIEE conference.

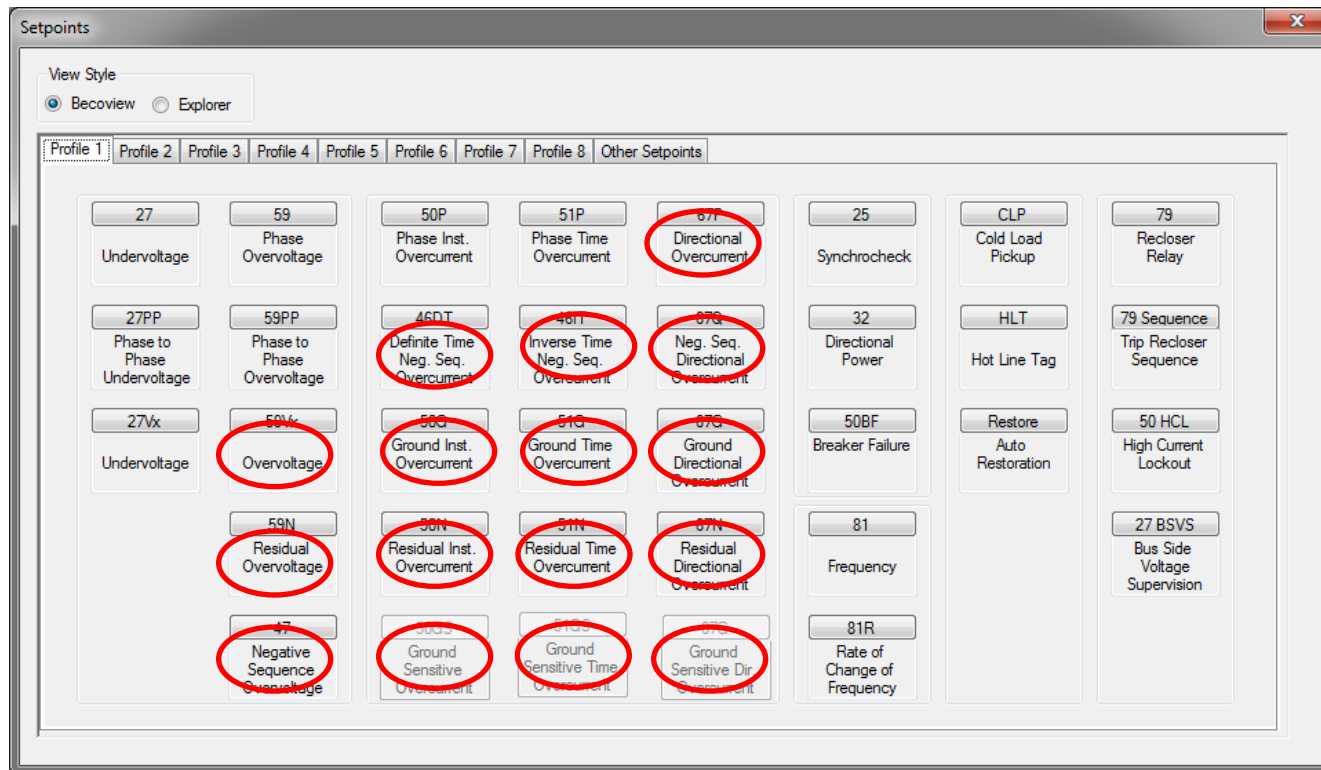
Symmetrical Components

Purpose:

From the relay's measured voltages and currents, symmetrical component math is used to calculate sequence components.

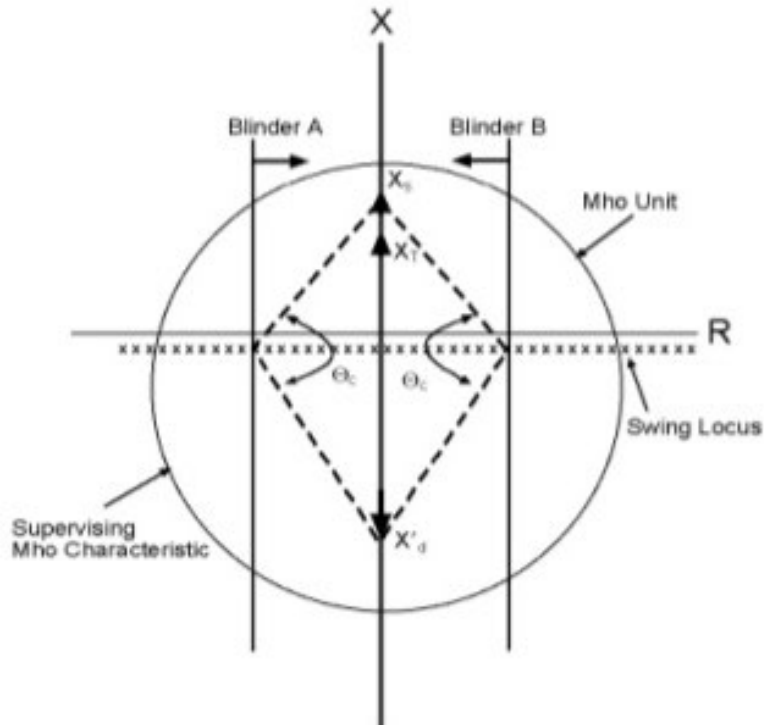
Why it is important to understand symmetrical components:

Because multi-function numerical relays use symmetrical component voltages and currents for some of the protection functions:

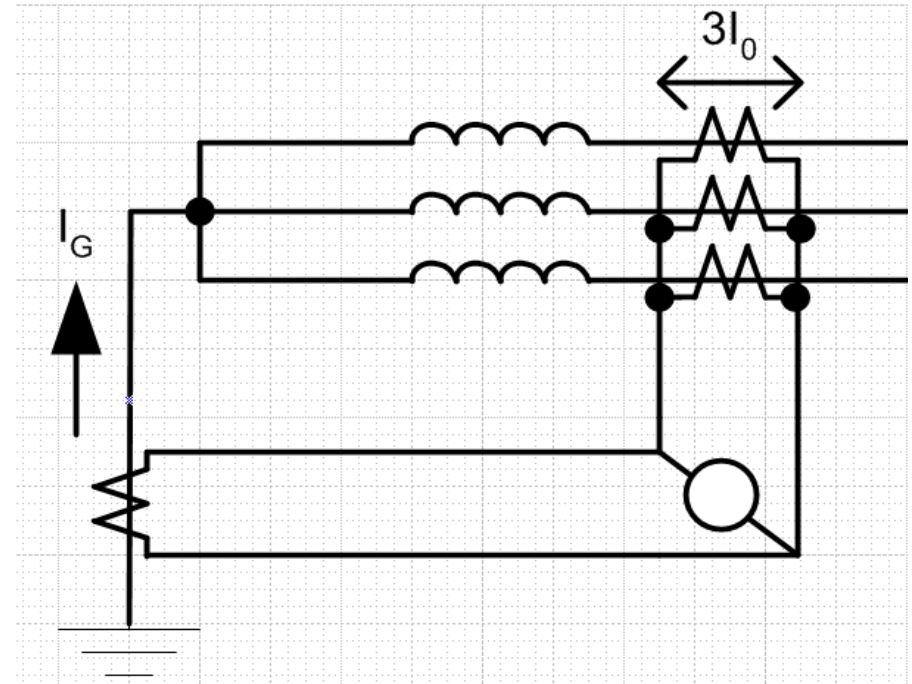


Symmetrical Components

Examples of Protection functions that use symmetrical component quantities:



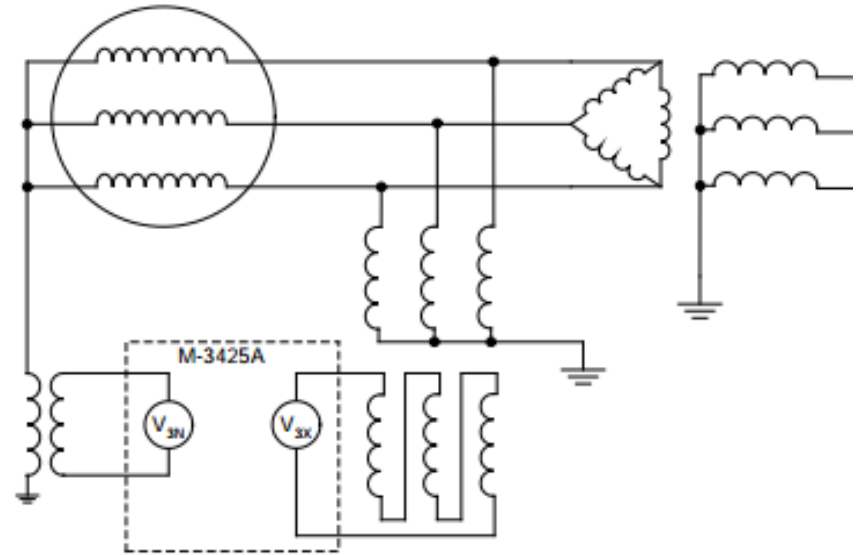
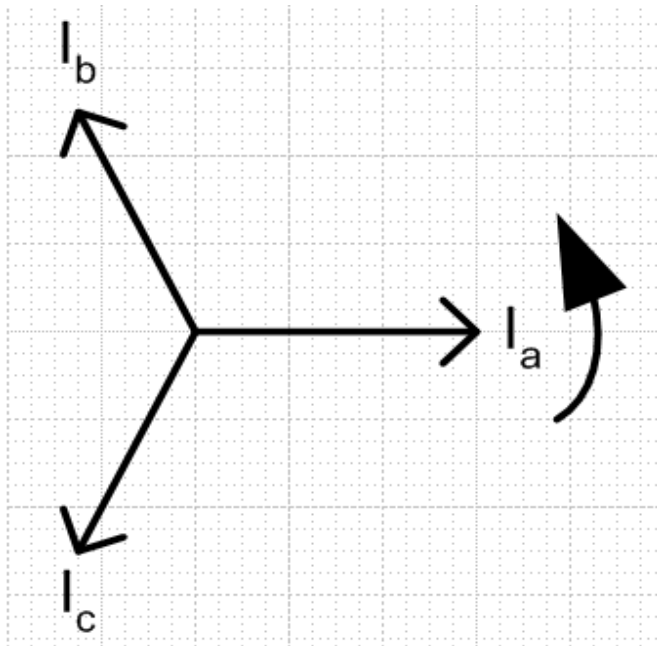
78 – Out-of-Step
(Positive-Sequence Impedance)



87GD – Ground Differential
(Zero-Sequence Current)

Symmetrical Components

Examples of Protection functions that use symmetrical component quantities:
(Continued)



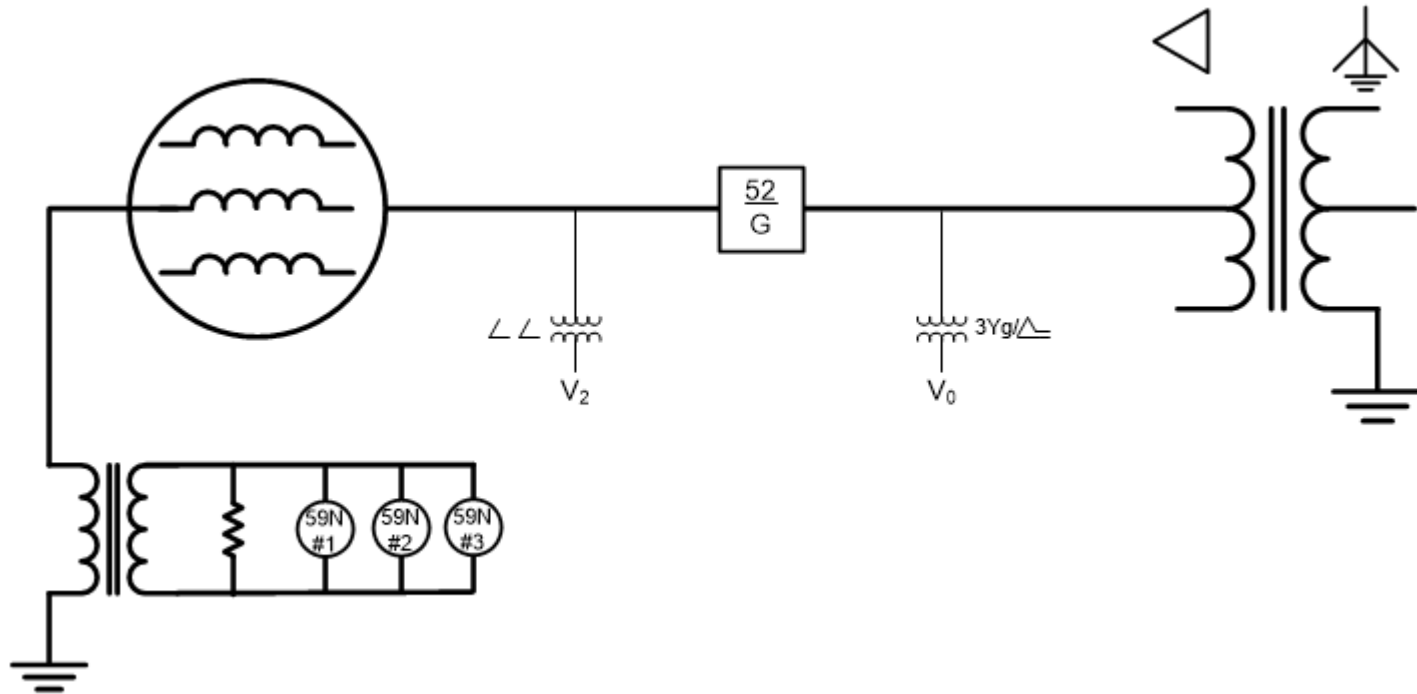
$$\text{The ratio } \left(\frac{V_x^{3rd}}{V_N^{3rd}} \right) \text{ OR } \left(\frac{3V_o^{3rd}}{V_N^{3rd}} \right) > \text{Pickup}$$

46 – Negative-Sequence Overcurrent

59D – Third Harmonic Voltage Ratio
(Zero-Sequence Voltage)

Symmetrical Components

Examples of Protection functions that use symmetrical component quantities:
(Continued)



59N – accelerated ground overvoltage scheme with V_2 and V_0 supervision

- $V_0 > V_2$ for GSU low side ground faults
- $V_0 < V_2$ for GSU high side ground faults

Symmetrical Components

- Each phase of voltage and current is made up of 3 separate components (or sequences) such that the sum of these components make up the total:

$$IA = IA0 + IA1 + IA2$$

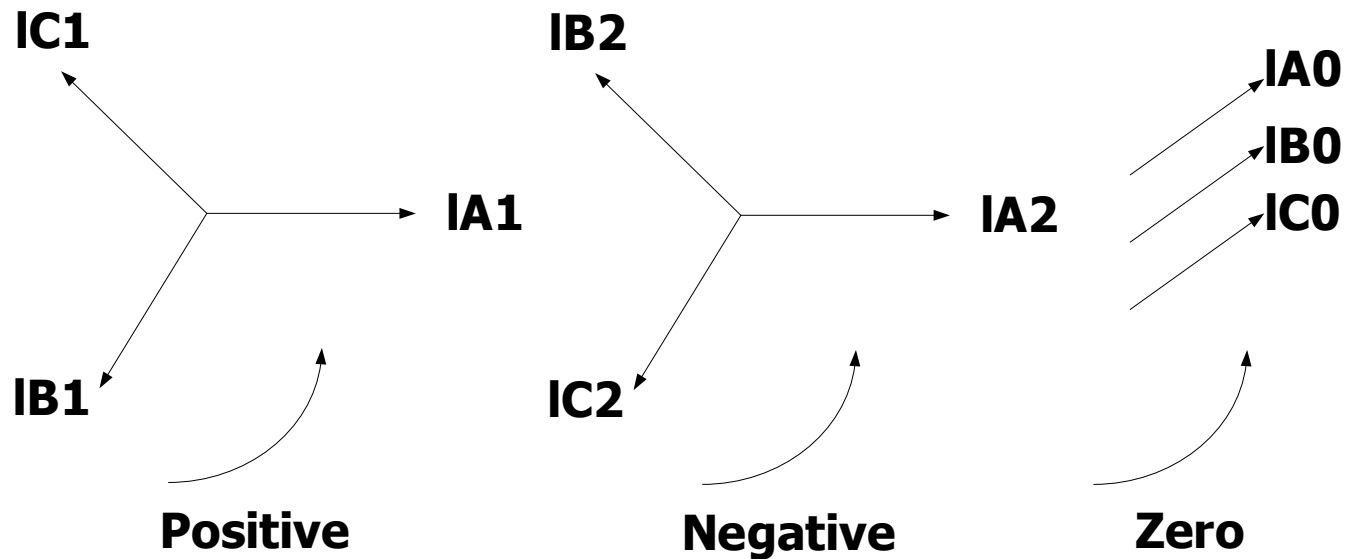
$$IB = IB0 + IB1 + IB2$$

$$IC = IC0 + IC1 + IC2$$

- Although the phases are unbalanced, the individual sequence networks are balanced.
- This allows us to go from 6 degrees of freedom to just 2.
- Any individual sequence component can never exist alone in 1 phase i.e. if any sequence is in 1 phase, then it must exist in all 3 phases.
- And each sequence component must be equal in magnitude in all 3 phases.
- $IA0$, $IB0$, and $IC0$ also have equal phase angles.
- While $IA1$, $IB1$, $IC1$ and $IA2$, $IB2$, $IC2$ do not have equal phase angles, but they are always 120° apart as defined.

Symmetrical Components

Sequence Phasors



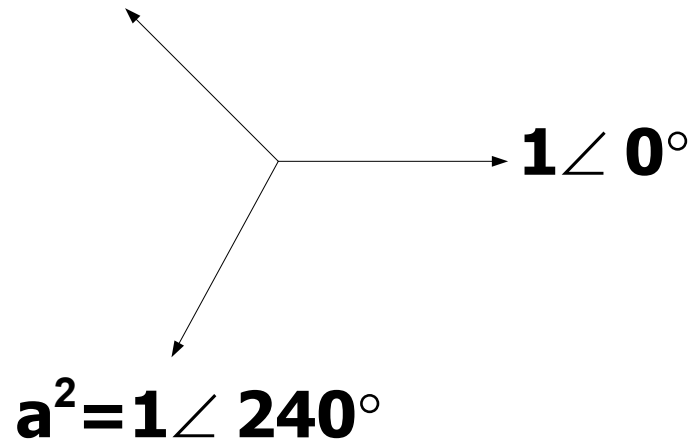
- Zero sequence phasors are of equal magnitude and in phase with each other.
- Zero sequence phasors do not rotate in sequence, but they do still rotate in time.

Symmetrical Components

“a” Operator or “a” Phasor

- Shorthand method for representing 2 of the phases (typically B and C), using the reference phase (typically A phase) shifted by the appropriate phase angle.
- “a” rotates a vector by 120°
- “a²” rotates a vector by 240°

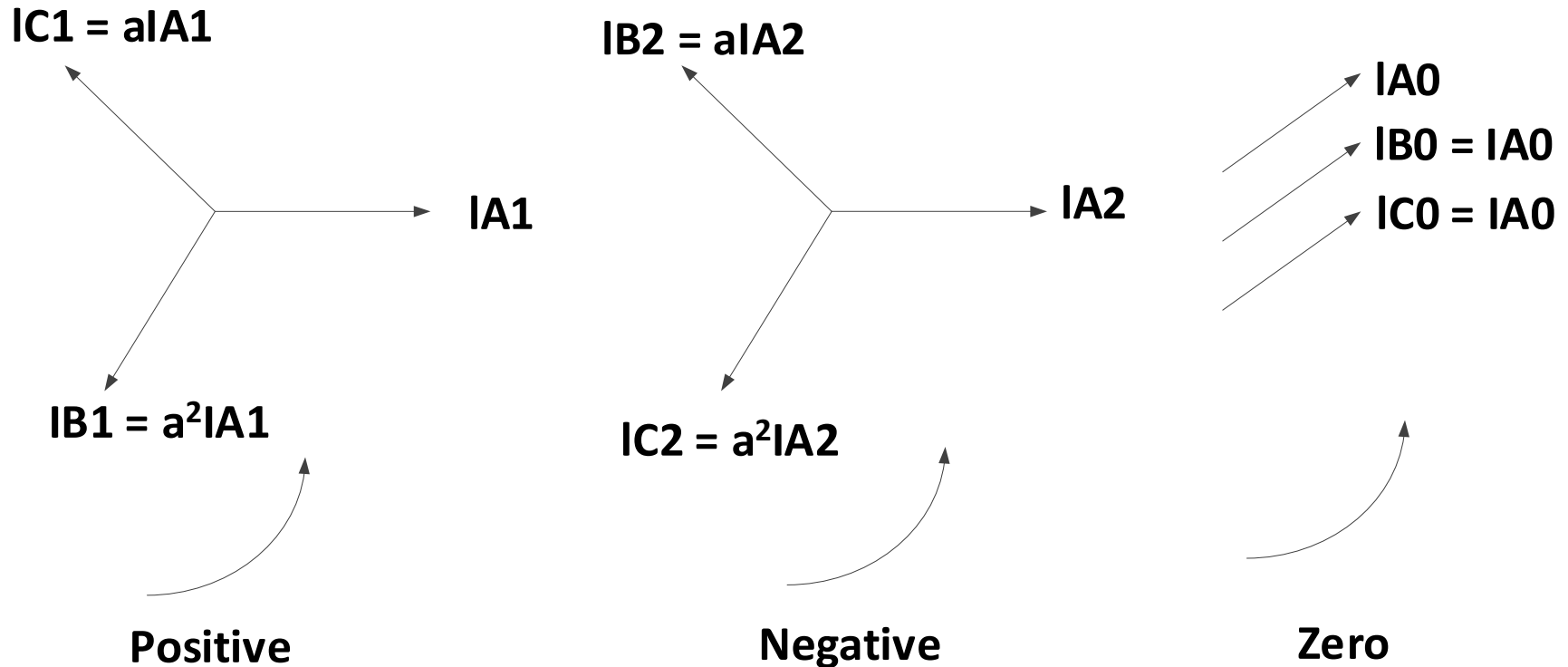
$$\mathbf{a} = \mathbf{1} \angle 120^\circ$$



Symmetrical Components

Combine Sequence Phasors with “a” Phasors

- To represent them all in terms of one phase (typically A-Phase)



Symmetrical Components

Sequence Network Equations

- Re-write the phase equations with “a” operators included
- If no phase designation, assume it is in reference to A phase:

$$I_A = I_{A0} + I_{A1} + I_{A2} = I_0 + I_1 + I_2$$

$$I_B = I_{B0} + I_{B1} + I_{B2} = I_{A0} + a^2 I_{A1} + a I_{A2} = I_0 + a^2 I_1 + a I_2$$

$$I_C = I_{C0} + I_{C1} + I_{C2} = I_{A0} + a I_{A1} + a^2 I_{A2} = I_0 + a I_1 + a^2 I_2$$

- Solve simultaneous equations for sequence quantities:

$$I_0 = 1/3 * (I_A + I_B + I_C)$$

$$I_1 = 1/3 * (I_A + a I_B + a^2 I_C)$$

$$I_2 = 1/3 * (I_A + a^2 I_B + a I_C)$$



$$3I_0 = I_A + I_B + I_C$$

$$3I_1 = I_A + a I_B + a^2 I_C$$

$$3I_2 = I_A + a^2 I_B + a I_C$$

- Negative-sequence and zero-sequence quantities typically only exist during system unbalance or unbalanced faults. Therefore, protection functions that operate on these quantities can be set to be more sensitive.

Same equations for voltage or current

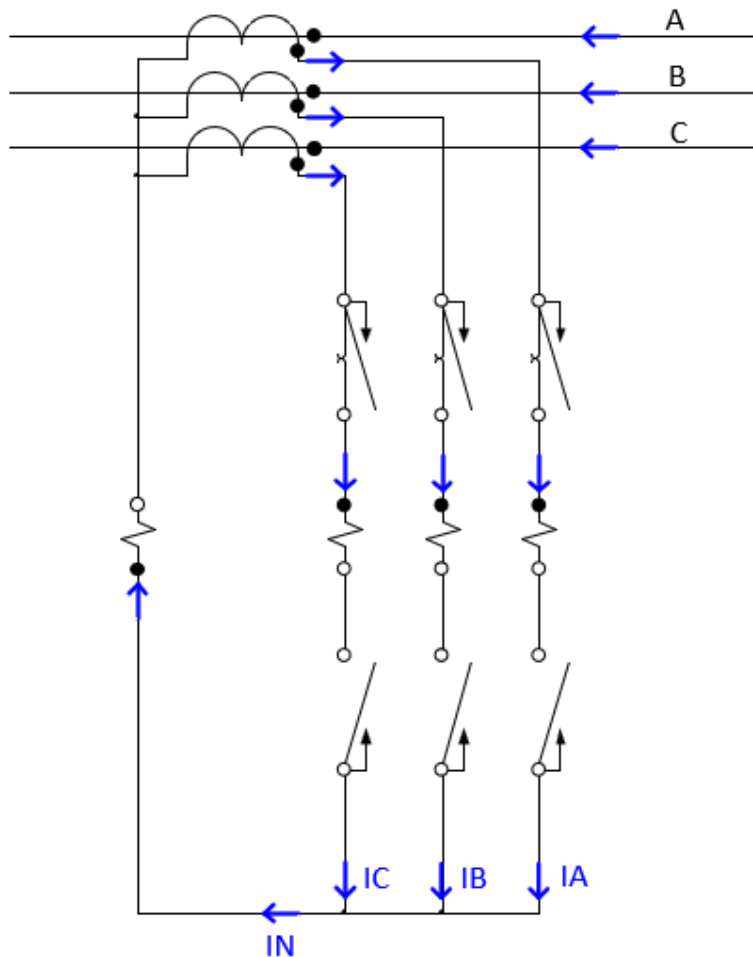
Symmetrical Components

$I_N = 3I_0$ proof:

$$I_N = I_A + I_B + I_C$$

$$I_N = (I_{A0} + I_{A1} + I_{A2}) + (I_{A0} + a^2 I_{A1} + a I_{A2}) + (I_{A0} + a I_{A1} + a^2 I_{A2})$$

$$I_N = 3I_{A0} + I_{A1}(1 + a + a^2) + I_{A2}(1 + a + a^2)$$



$$(1 + a + a^2) = 0$$

$$a = 1 \angle 120^\circ$$

$$a^2 = 1 \angle 240^\circ$$

$$1 \angle 0^\circ$$

$$a^2 = 1 \angle 240^\circ$$

$$a = 1 \angle 120^\circ$$

$$1 \angle 0^\circ$$

$$I_N = 3I_{A0}$$

$$I_N = 3I_0$$

$$3I_0 = I_A + I_B + I_C$$

Symmetrical Components

Example Calculation using sequence equations

- Given the following unbalanced currents:

$$I_a = 148.7 \angle 3.3^\circ$$

$$I_b = 49.3 \angle 142.3^\circ$$

$$I_c = 41.2 \angle 198.6^\circ$$

- Calculate the symmetrical component values using the equations:

$$I_{a0} = 1/3 * (I_a + I_b + I_c) = 25 \angle 20^\circ$$

$$I_{a1} = 1/3 * (I_a + aI_b + a^2I_c) = 50 \angle 0^\circ$$

$$I_{a2} = 1/3 * (I_a + a^2I_b + aI_c) = 75 \angle 0^\circ$$

$$I_{b0} = I_{a0} = 25 \angle 20^\circ$$

$$I_{b1} = a^2I_{a1} = 50 \angle 240^\circ$$

$$I_{b2} = aI_{a2} = 75 \angle 120^\circ$$

$$I_{c0} = I_{a0} = 25 \angle 20^\circ$$

$$I_{c1} = aI_{a1} = 50 \angle 120^\circ$$

$$I_{c2} = a^2I_{a2} = 75 \angle 240^\circ$$

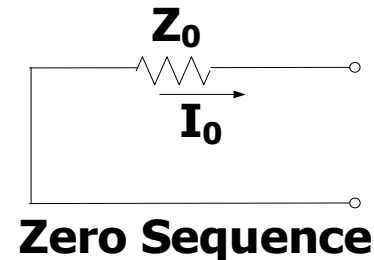
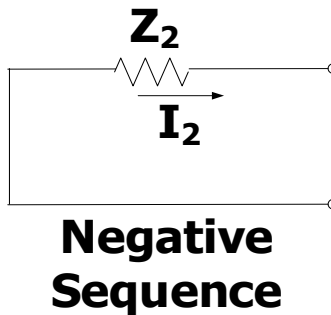
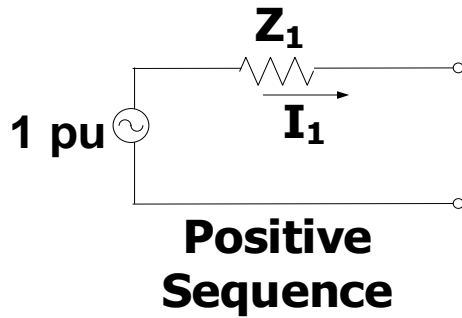
The screenshot shows a web browser interface for a symmetrical component calculator. The page title is "Phase to Symm Comp" and "Symm to Phase Comp". There are two tabs: "Phase to Symm Comp" (selected) and "Symm to Phase Comp". Under "Phase Rotation", there are buttons for "ABC" and "ACB". Under "Units", there are buttons for "V" and "I". Under "Phase Values", there are two columns: "Magnitud" and "Angle". The input values are: Ia = 148.7, Ib = 49.3, Ic = 41.2; and Angles = 3.3, 142.3, 198.6. Under "Symmetrical Components", there are three rows: I0 (24.97, 19.96), I1 (50.00, 0.00), and I2 (74.99, 0.00). At the bottom, there are buttons for "Base" and "A", "a", "c".

http://www.relaytech.com/symmcomp_calculator.htm

Symmetrical Components

For short circuit calculations to connect the sequence networks for different conditions, must know how to represent different equipment:

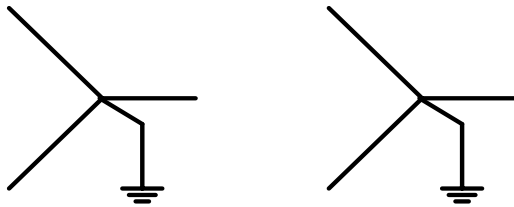
Symmetrical Components Network Representations



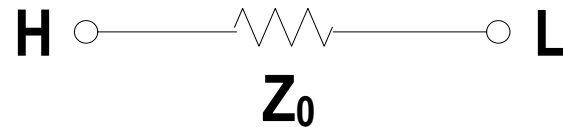
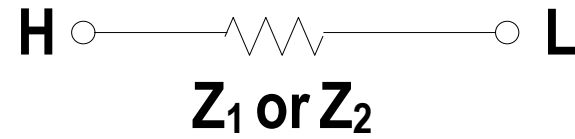
Symmetrical Components

Short Circuit Calculations

Symmetrical Components Transformer Representations



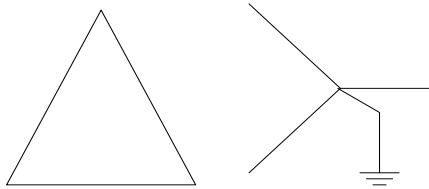
Grounded Wye - Grounded Wye



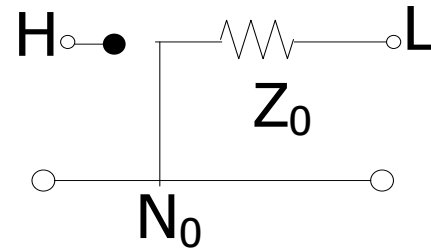
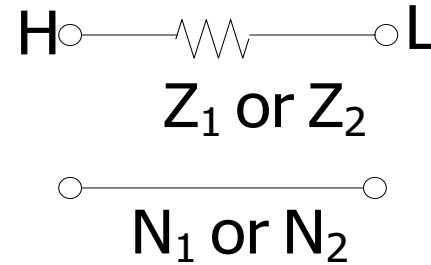
Symmetrical Components

Short Circuit Calculations

Symmetrical Components Transformer Representations



Delta- Grounded Wye



Fault Types

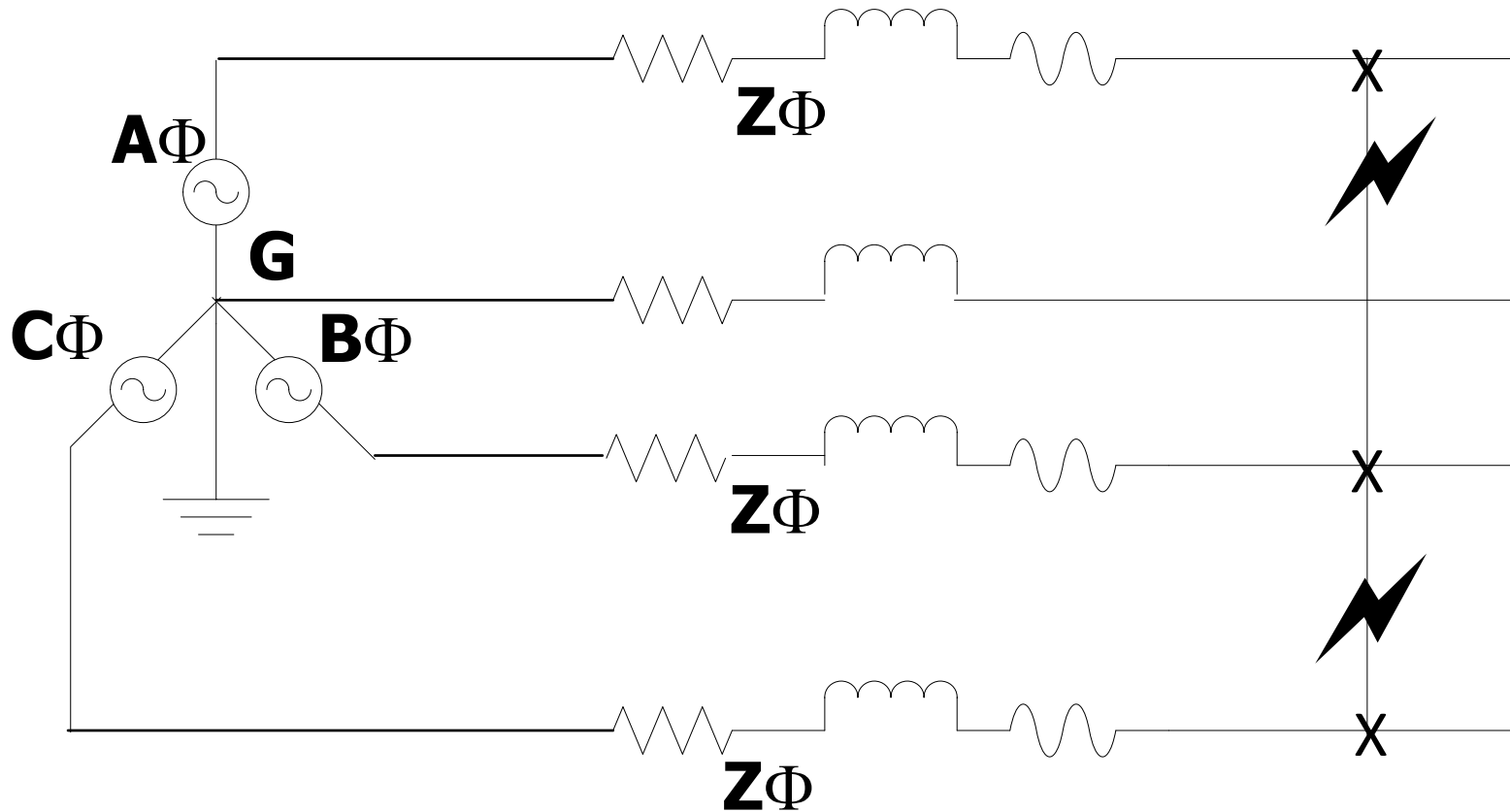
Most faults are ΦG faults:

- 77% of faults are ΦG
- 12% of faults are $\Phi\Phi$
- 7% of faults are $\Phi\Phi G$
- 4% of faults are 3Φ

<u>Fault</u>	<u>Sequence network involved</u>
3Φ	positive
$\Phi\Phi$	positive, negative
ΦG	positive, negative, zero
$\Phi\Phi G$	positive, negative, zero

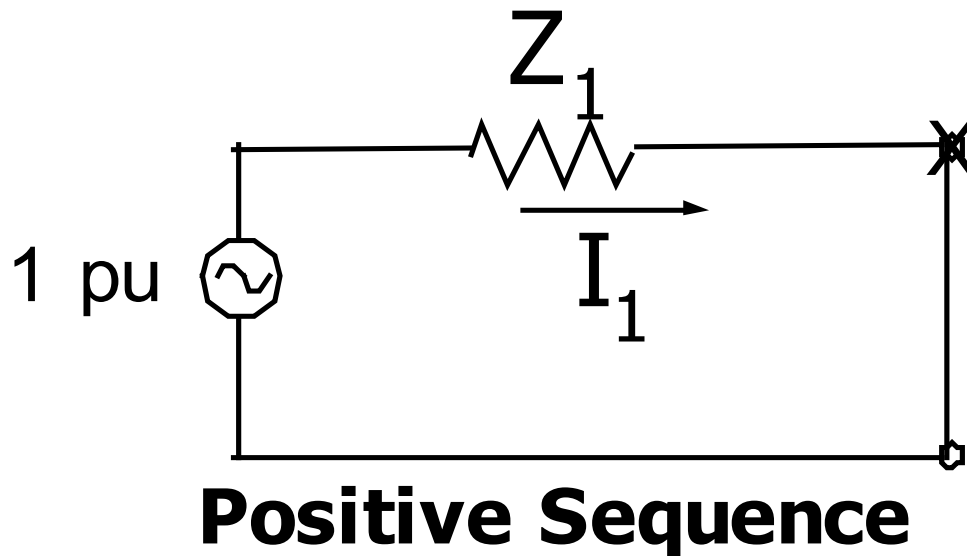
Fault Types

3 Φ Fault – represented on a 3 Phase Diagram



Fault Types

3 Φ Fault – Sequence Network



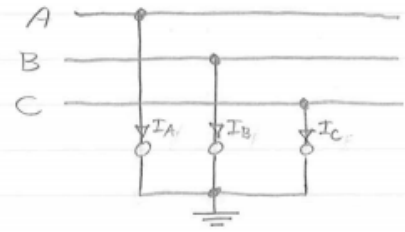
Question: Why is the sequence network for a 3 Φ fault represented with this circuit connection and with no negative or zero sequence circuits?

Fault Types

Answer: Identify Boundary Conditions and solve sequence equations.

3 ϕ FAULT (SAME AS 3 ϕ -GND FAULT)

only pos seq



B.C.

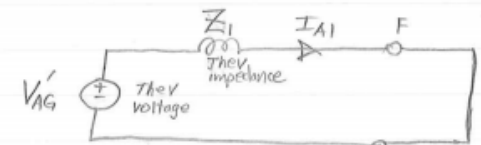
- ① $I_A = I_{AF}$
- ② $I_A + I_B + I_C = 0$
- ③ $V_{AG} = V_{BG} = V_{CG} = 0$
- ④ $I_C = a I_A$
 $I_B = a^2 I_A$

SOLVE

- ② $I_{A0} = \frac{1}{3}(I_A + I_B + I_C) \Rightarrow \underline{I_{A0} = 0}$
 $I_{A1} = \frac{1}{3}(I_A + a I_B + a^2 I_C)$
- ④ $= \frac{1}{3}(I_A + a^3 I_A + a^3 I_A)$
- ① $= \frac{1}{3}(I_A + I_A + I_A) \Rightarrow \underline{I_{A1} = I_{AF} = I_B = I_C}$
- ④ $I_{A2} = \frac{1}{3}(I_A + a^2 I_B + a I_C)$
 $= \frac{1}{3}(I_A + a^4 I_A + a^2 I_A)$
 $= \frac{1}{3} I_A (1 + a + a^2) \Rightarrow \underline{I_{A2} = 0}$

- ③ $V_{A0} = \frac{1}{3}(V_{AG} + V_{BG} + V_{CG})$
 $= \frac{1}{3}(V_{AG} + V_{AG} + V_{AG}) \Rightarrow \underline{V_{A0} = 0}$
- ③ $V_{A1} = \frac{1}{3}(V_{AG} + a V_{BG} + a^2 V_{CG}) \Rightarrow \underline{V_{A1} = 0}$
- ③ $V_{A2} = \frac{1}{3}(V_{AG} + a^2 V_{BG} + a V_{CG}) \Rightarrow \underline{V_{A2} = 0}$

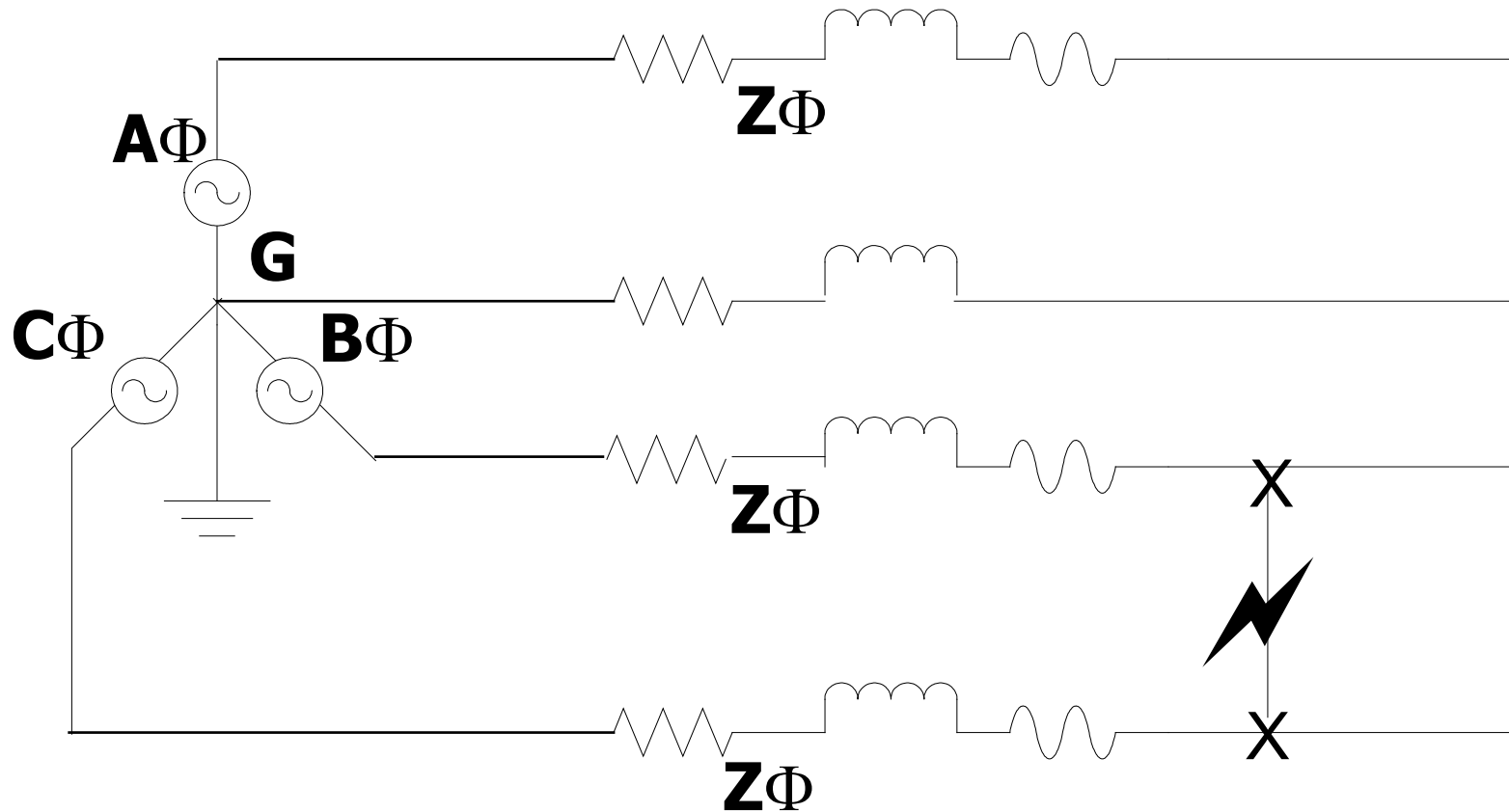
SEQUENCE NETWORK



$I_{A1} = \frac{V'_{AG}}{Z_1}$

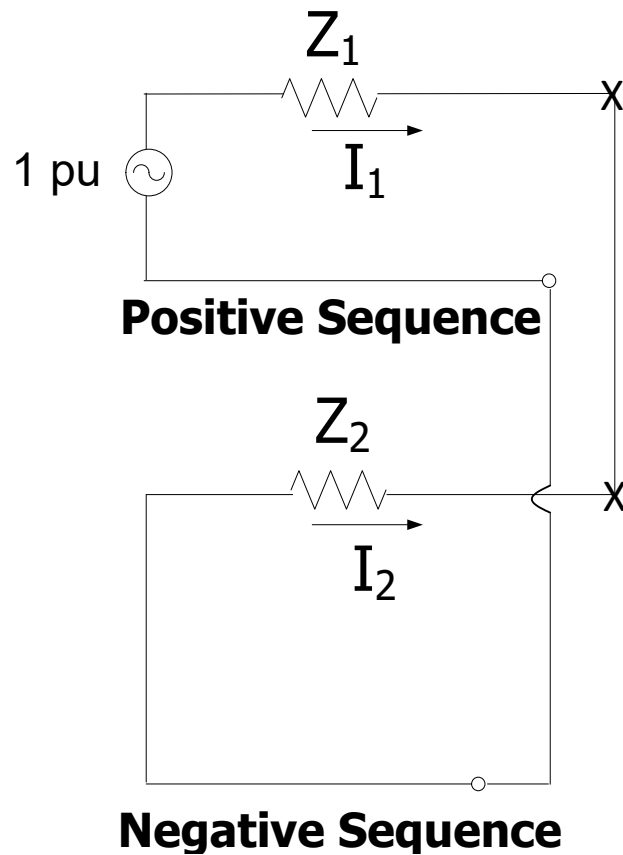
Fault Types

$\Phi\Phi$ Fault – represented on a 3 Phase Diagram



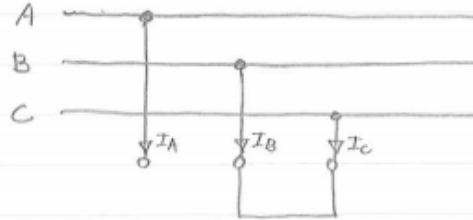
Fault Types

$\Phi\Phi$ Fault – Sequence Networks



Fault Types

B-C FAULT



B.C.

- ① $I_B = -I_C$
- ② $I_A = 0$
- ③ $V_{BG} = V_{CG}$

$$I_{A0} = \frac{1}{3}(I_A + I_B + I_C)$$

① & ② $I_{A0} = \frac{1}{3}(0 + I_B - I_B) \Rightarrow \underline{I_{A0} = 0}$

① $I_B = -I_C$

$$I_{A0} + a^2 I_{A1} + a I_{A2} = -I_{A0} - a I_{A1} - a^2 I_{A2}$$

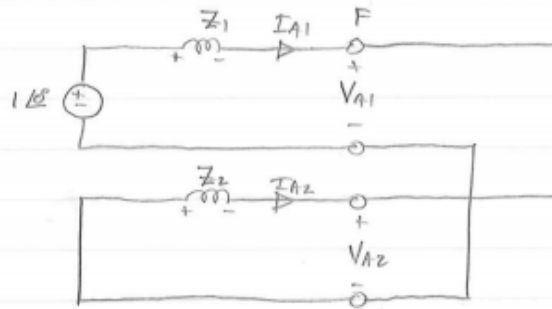
$$(a^2 + a) I_{A1} = -(a^2 + a) I_{A2} \Rightarrow \underline{I_{A1} = -I_{A2}}$$

$$V_{A1} = \frac{1}{3}(V_{AG} + a V_{BG} + a^2 V_{CG})$$

③ $V_{A1} = \frac{1}{3}(V_{AG} + a V_{BG} + a^2 V_{BG})$ ④

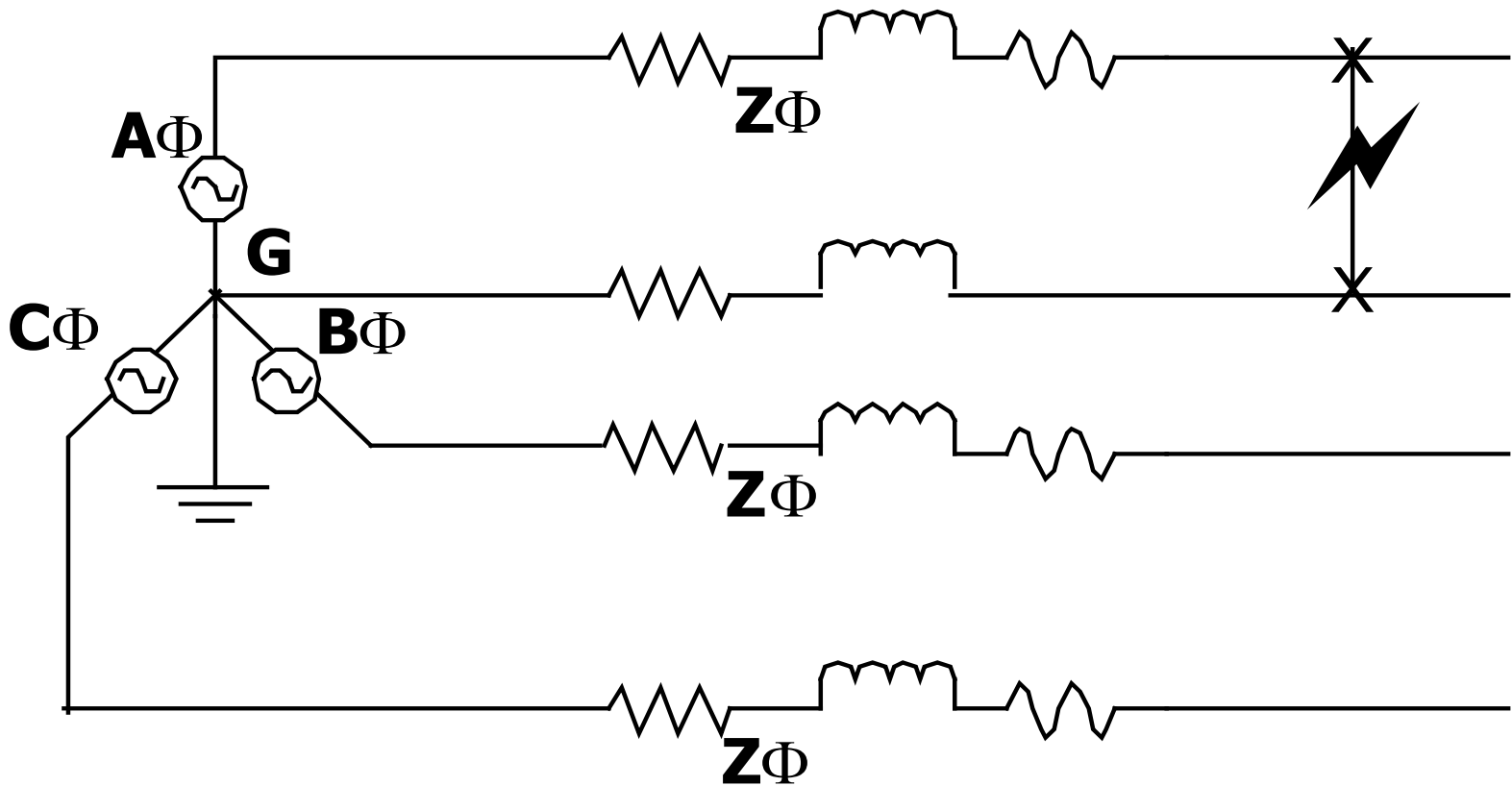
$$V_{A2} = \frac{1}{3}(V_{AG} + a^2 V_{BG} + a V_{CG})$$

③ $V_{A2} = \frac{1}{3}(V_{AG} + a^2 V_{BG} + a V_{BG})$ ⑤ $\Rightarrow \underline{V_{A1} = V_{A2}}$



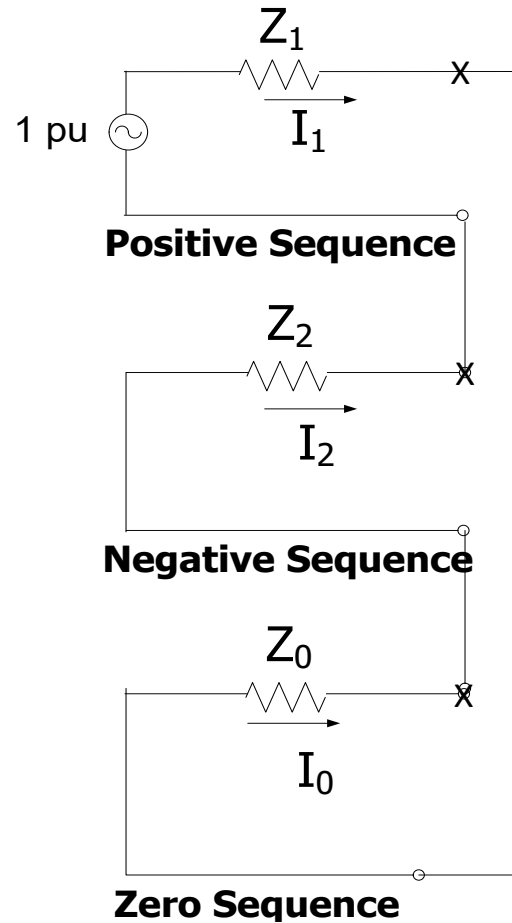
Fault Types

ΦG Fault – represented on a 3 Phase Diagram



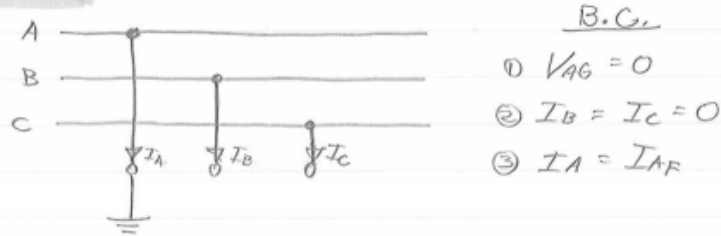
Fault Types

Φ G Fault – Sequence Networks

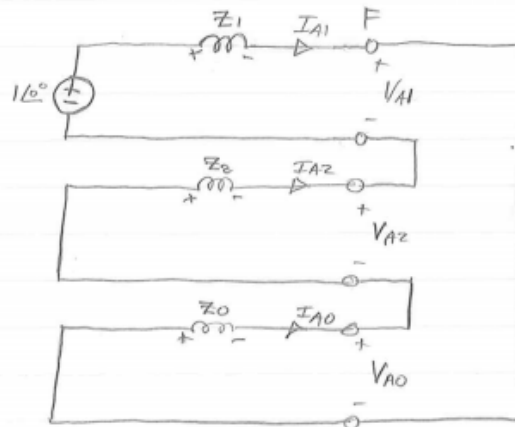


Fault Types

A-GND FAULT



- ① $V_{AG} = 0 \Rightarrow \underline{V_{A0} + V_{A1} + V_{A2} = 0}$
- ② $I_B = I_C = 0$
 $I_{A0} + a^2 I_{A1} + a I_{A2} = I_{A0} + a I_{A1} + a^2 I_{A2}$
 $(a^2 - a) I_{A1} = (a^2 - a) I_{A2} \Rightarrow I_{A1} = I_{A2}$ ④
- ② $I_B = 0$
 $I_{A0} + a^2 I_{A1} + a I_{A2} = 0$
- ④ $I_{A0} + a^2 I_{A1} + a I_{A1} = 0$
 $I_{A0} = -(a^2 + a) I_{A1}$ $1 + a + a^2 = 0 \Rightarrow 1 = -(a + a^2)$
 $I_{A0} = I_{A1} \Rightarrow \underline{I_{A0} = I_{A1} = I_{A2}}$



$$1 = Z_1 I_{A1} + V_{A1}$$

$$V_{A1} = 1 - Z_1 I_{A1}$$

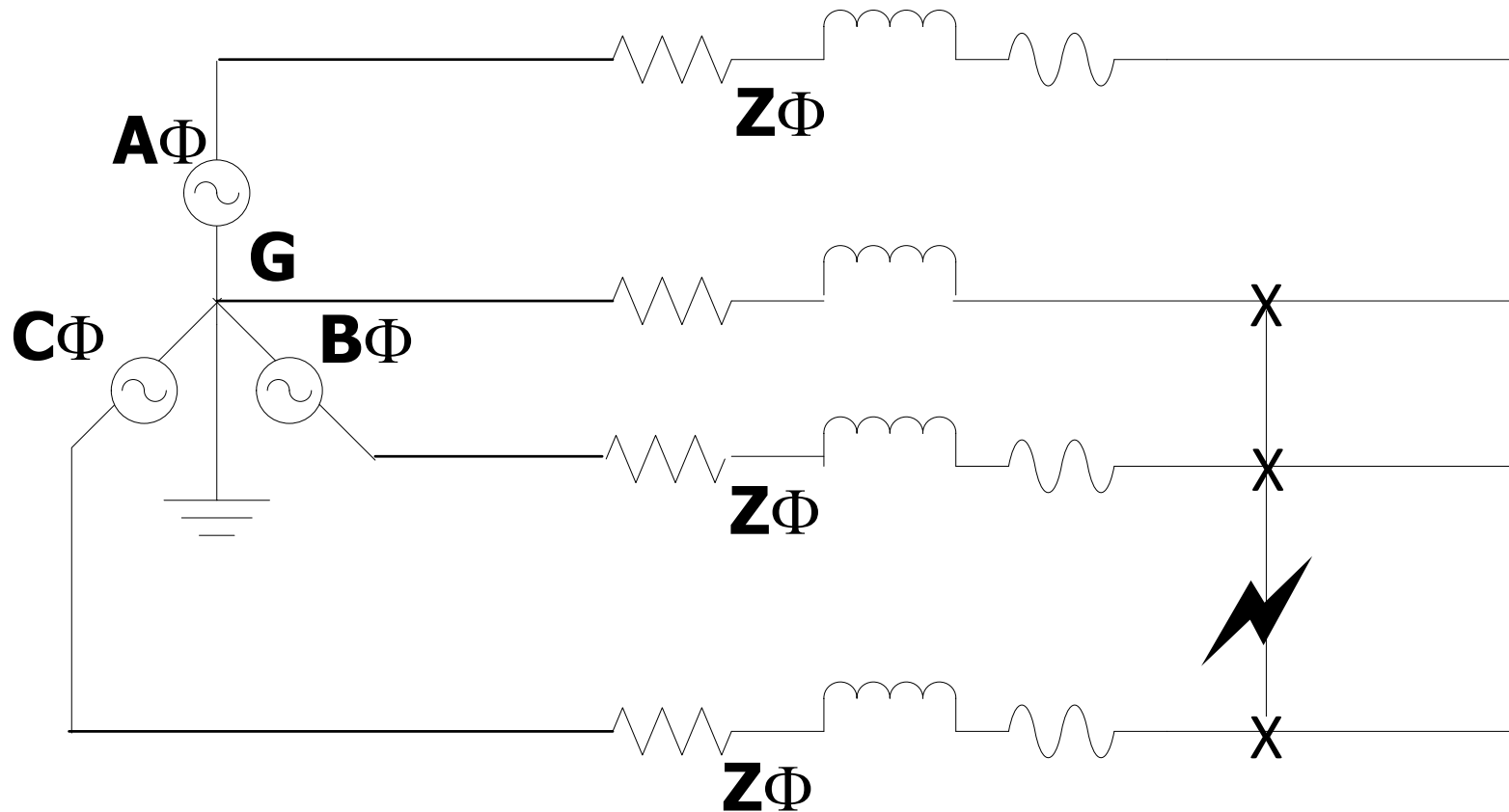
$$0 = Z_2 I_{A2} + V_{A2}$$

$$V_{A2} = -Z_2 I_{A2}$$

$$V_{A0} = -Z_0 I_{A0}$$

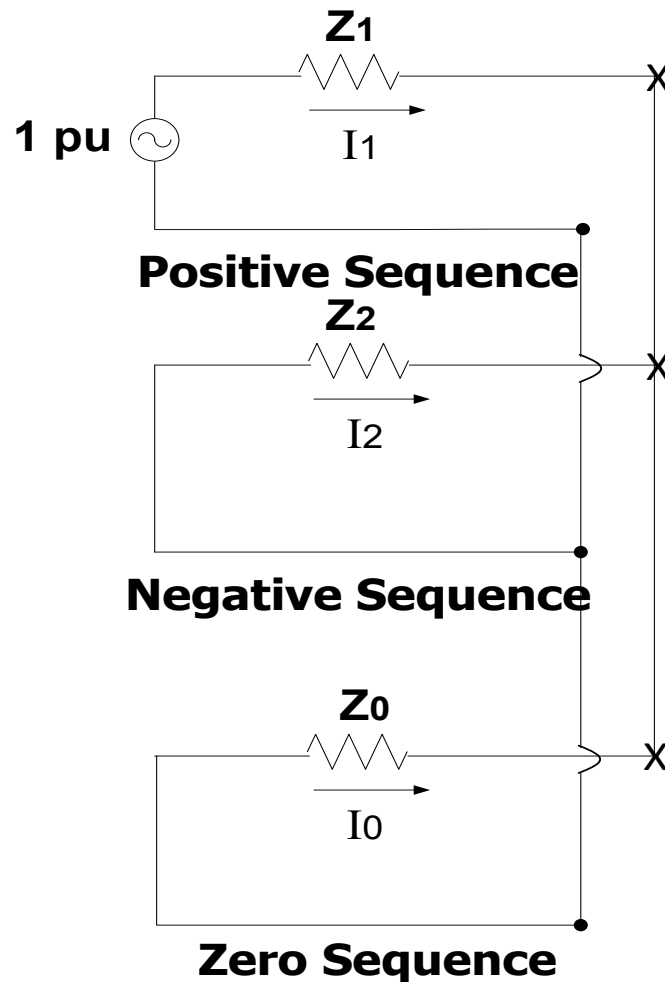
Fault Types

$\Phi\Phi G$ Fault – represented on a 3 Phase Diagram



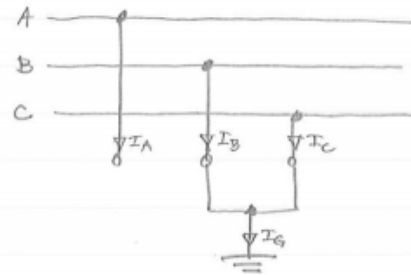
Fault Types

$\Phi\Phi G$ Fault – Sequence Networks



Fault Types

B-C-GND FAULT



B.C.

- ① $I_A = 0$
- ② $V_{BG} = V_{CG} = 0$

① $I_A = 0 \Rightarrow \underline{I_{A0} + I_{A1} + I_{A2} = 0}$

$V_{A0} = \frac{1}{3} (V_{AG} + V_{BG} + V_{CG})$

② $V_{A0} = \frac{1}{3} V_{AG}$ ③

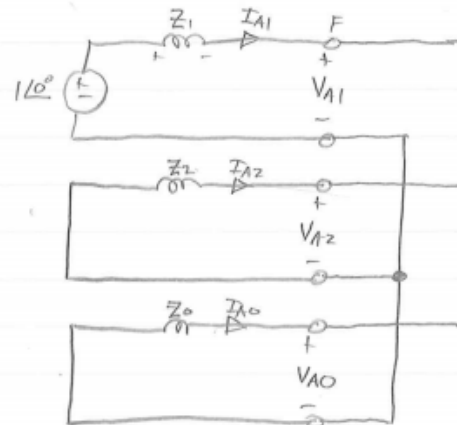
$V_{A1} = \frac{1}{3} (V_{AG} + a V_{BG} + a^2 V_{CG})$

④ $V_{A1} = \frac{1}{3} V_{AG}$ ④

$V_{A2} = \frac{1}{3} (V_{AG} + a^2 V_{BG} + a V_{CG})$

⑤ $V_{A2} = \frac{1}{3} V_{AG}$ ⑤

$\therefore \text{③} = \text{④} = \text{⑤} \Rightarrow \underline{V_{A0} = V_{A1} = V_{A2}}$



$V_{A1} = -Z_1 I_{A1}$

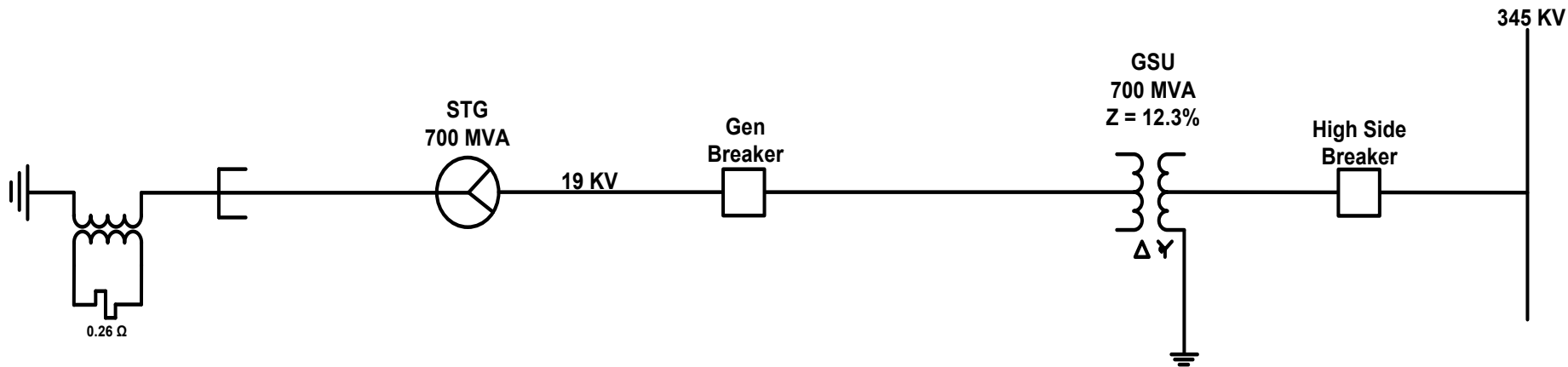
$V_{A2} = -Z_2 I_{A2}$

$V_{A0} = -Z_0 I_{A0}$

Fault Calculation Examples

Generator Fault Calculations for (5) Cases:

- 1) 3 Φ fault on LS of GSU
 - 2) 3 Φ fault on HS of GSU
 - 3) Φ G fault on LS of GSU
 - 4) Φ G fault on HS of GSU
 - 5) V2 & V0 calcs for LS & HS Φ G fault for 59N supervision
- *LS = Low Voltage Side*
 - *HS = High Voltage Side*



Fault Calculation Examples

Generator, NGR, GSU, and System Thevenin Data

- All values in per unit at generator base of 700 MVA.
- Ignore generator, GSU, and system thevenin resistances for ease of calculations.

	<u>Saturated</u>	<u>Unsaturated</u>	<u>Time Constants</u>	
X_d''	0.224	0.260	$T_d'' = 0.025$ seconds	$T_{do}'' = 0.031$ seconds
X_d'	0.295	0.324	$T_d' = 0.9$ seconds (3 Φ short-circuit)	$T_{do}' = 5.1$ seconds
X_d	1.66	1.83	$T_d = 1.4$ seconds ($\Phi\Phi$ short-circuit)	
X_{g2}	0.224	0.260	$T_d = 1.6$ seconds (ΦG short-circuit)	
X_{g0}	-	0.128		

Note: $X_{g2} = X_d''$ for nonsalient machines. This is a round rotor machine.

$$R_{gpri} = NGR_{pri} = NGR_{sec} * \left(\frac{NGT_{pri}}{NGT_{sec}} \right)^2 = 0.26 * \left(\frac{19000}{208} \right)^2 = 2169.47 \Omega_{pri}$$

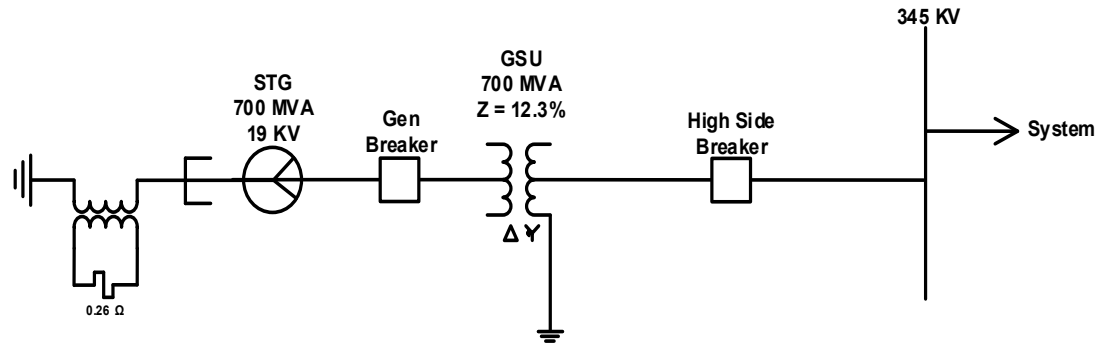
$$R_g \text{ limits ground current to } 3I_o = \frac{3 * V}{X_{g1} + X_{g2} + X_{g0} + 3R_g} = \frac{3V}{3R_g} = \frac{V}{R_g} = \frac{19000}{\sqrt{3} * 2169.47} = 5.1 A_{pri}$$

$$R_{gpu} = NGR_{pu} = \frac{NGR_{pri}}{Z_{baseLS}} = \frac{2169.47}{\frac{19^2}{700}} = 4206.7 pu$$

$$X_t = 0.123 pu$$

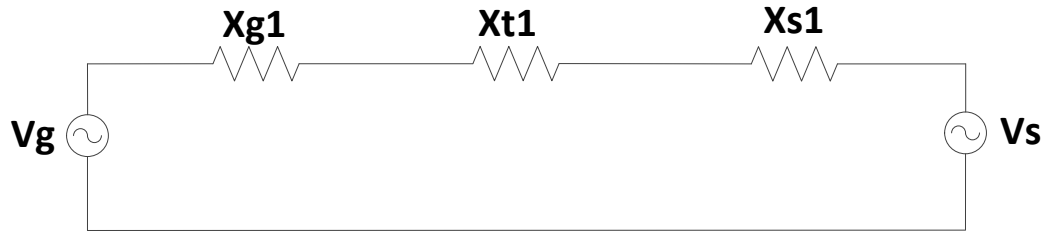
$$X_{s1} = 0.104 pu, \quad X_{s2} = 0.104 pu, \quad X_{s0} = 0.093 pu$$

Fault Calculation Examples

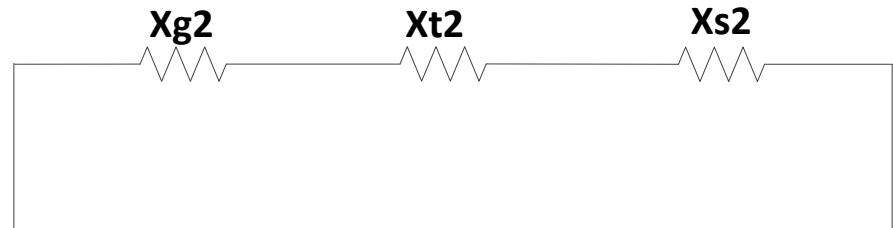


**Draw
Sequence Networks**

**Positive
Sequence**

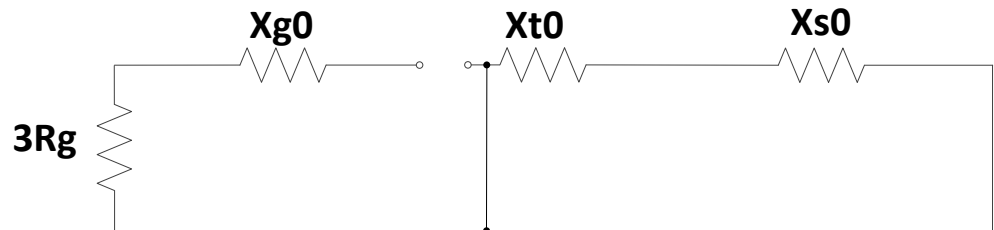


**Negative
Sequence**



Why 3R_g?

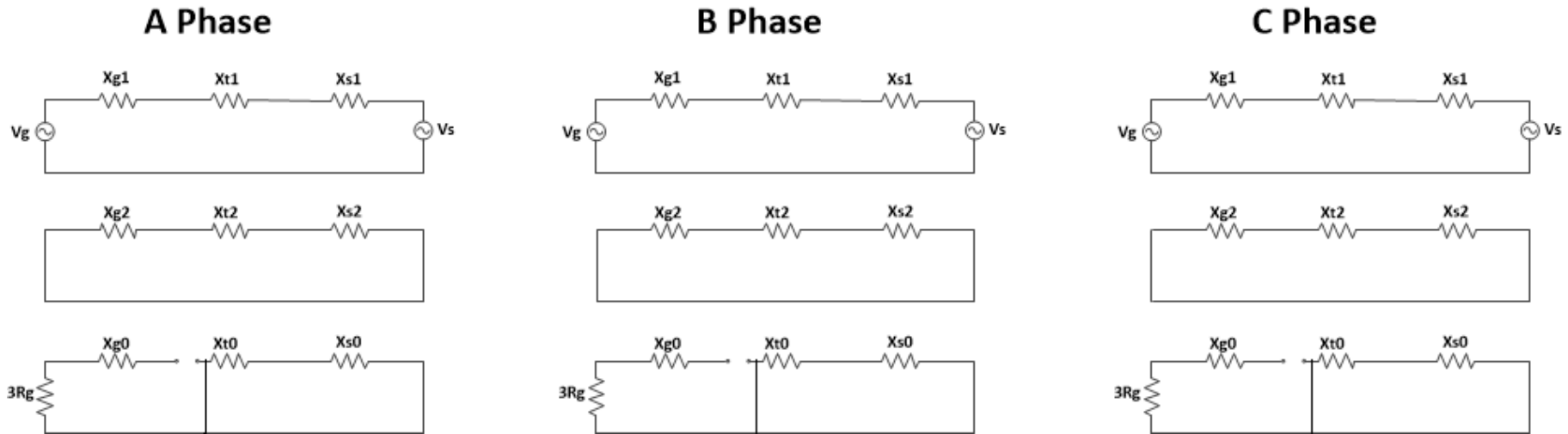
**Zero
Sequence**



Fault Calculation Examples

Why do we multiply R_g (NGR) by 3?

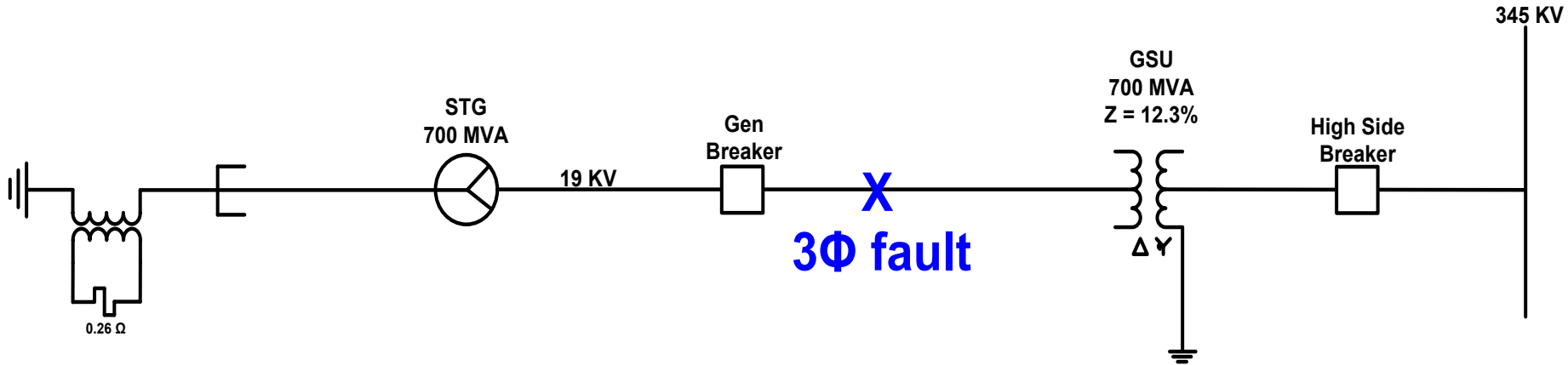
- The zero-sequence network represents 1 of the 3 phases where $1\text{pu } I_0$ flows:



- With equal current in all 3 phases, I_0 adds to 3pu current that is flowing up the ground and thru the NGR.
- Therefore, the voltage drop across the NGR in the zero-sequence network is $V_{NGR} = 3I_0 \cdot R_g$ (or if it is re-arranged $V_{NGR} = 3R_g \cdot I_0$).
- This means that the impedance of anything connected between the power system neutral (or star point) and ground must be multiplied by 3.**

Fault Calculation Examples

Case 1) 3 Φ fault on LS of GSU



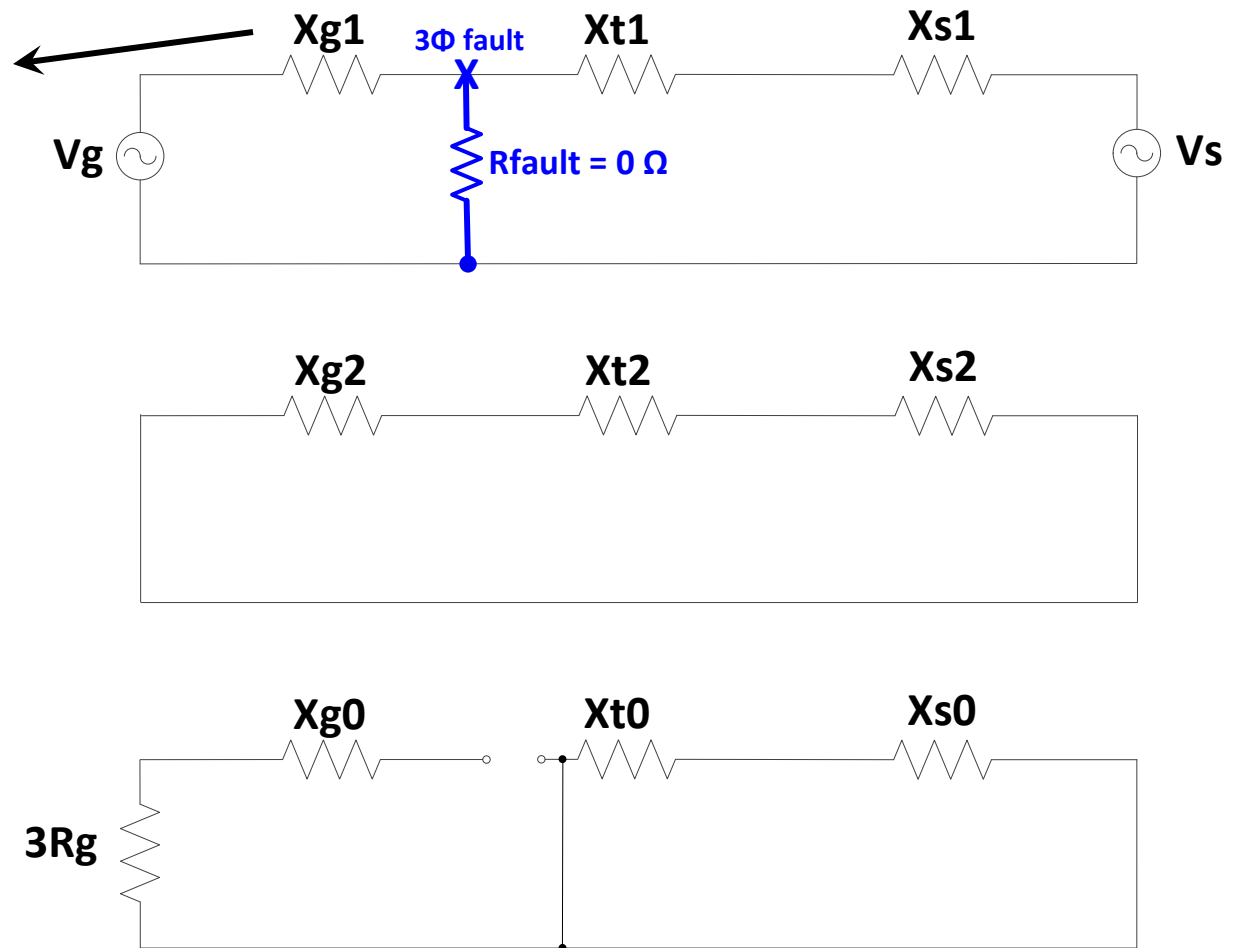
Fault Calculation Examples

3 phase fault on low side of GSU

- Assume no fault impedance, $R_{\text{fault}} = 0 \Omega$
- Only positive sequence network is involved for a balanced 3 phase fault
- No negative sequence or zero sequence circuit connections for this fault type

X_{g1} is 3 reactances,
decaying over time:

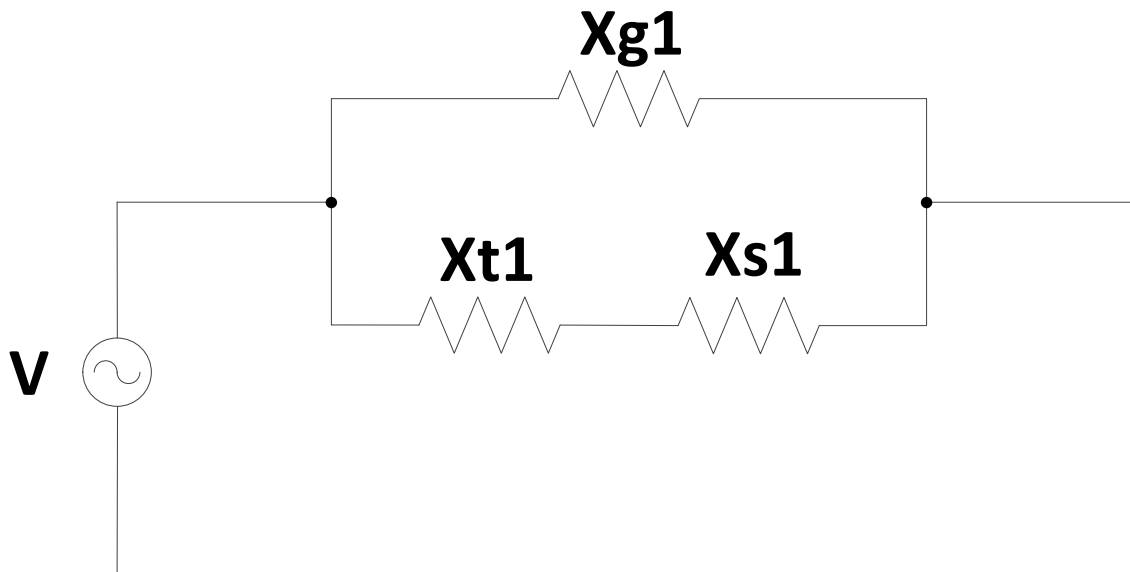
- $X_{d''}$
- $X_{d'}$
- X_d



Fault Calculation Examples

3 phase fault on low side of GSU – Subtransient

- First, calculate the subtransient fault current (use X_d'' for X_{g1}).
- Assume system voltage is at nominal value prior to the fault i.e. 1 pu.
- Assume all sources are in phase and of equal magnitude, which is equivalent to neglecting prefault load current.
- Therefore $V = V_g = V_s = 1$ pu at 0° , and network reduces as such:

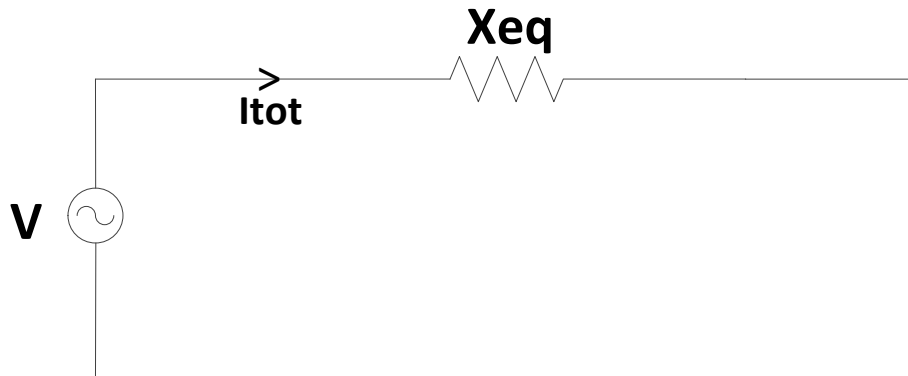


Fault Calculation Examples

3 phase fault on low side of GSU – Subtransient

- Calculate X_{eq} (using X_d'' for X_{g1}):

$$X_{eq} = \frac{X_{g1} * (X_{t1} + X_{s1})}{X_{g1} + X_{t1} + X_{s1}} = \frac{0.224 * (0.123 + 0.104)}{0.224 + 0.123 + 0.104} = 0.112745 \text{ pu}$$



- Calculate I_{tot} :

$$I_1 = I_{tot} = \frac{V}{X_{eq}} = \frac{1}{0.112745} = 8.86957 \text{ pu}$$

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_{a0} = I_{a2} = 0$$

$$I_a = I_{a1} = I_1 \quad I_b = a^2 I_1 \quad I_c = a I_1$$

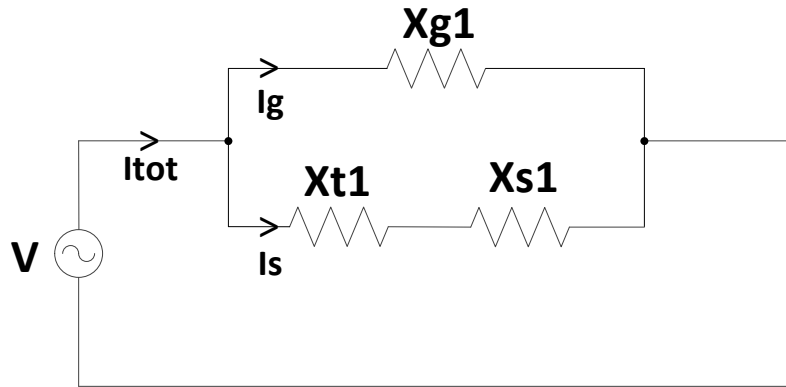
$$V_0 = V_2 = 0$$

$$V_1 = 0 = V_a = V_b = V_c$$

Fault Calculation Examples

3 phase fault on low side of GSU – Subtransient

- Use current division to calculate the gen and system contributions:



$$I_{g\text{pu}} = I_{tot} * \frac{X_{t1} + X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = 8.86957 * \frac{0.123 + 0.104}{0.224 + 0.123 + 0.104} = 4.464 \text{ pu}$$

$$I_{s\text{pu}} = I_{tot} * \frac{X_{g1}}{X_{g1} + X_{t1} + X_{s1}} = 8.86957 * \frac{0.224}{0.224 + 0.123 + 0.104} = 4.405 \text{ pu}$$

Fault Calculation Examples

3 phase fault on low side of GSU – Subtransient

- Convert from per unit to actual amps:

$$I_{actual} = I_{pu} * I_{base}$$

Generator contribution to the fault:

$$I_d'' = I_{gact}'' = I_{gpu}'' * I_{baseLS} = 4.464 * \frac{700 * 1000}{\sqrt{3} * 19} = 94,953 \text{ pri amps}$$

System contribution to the fault:

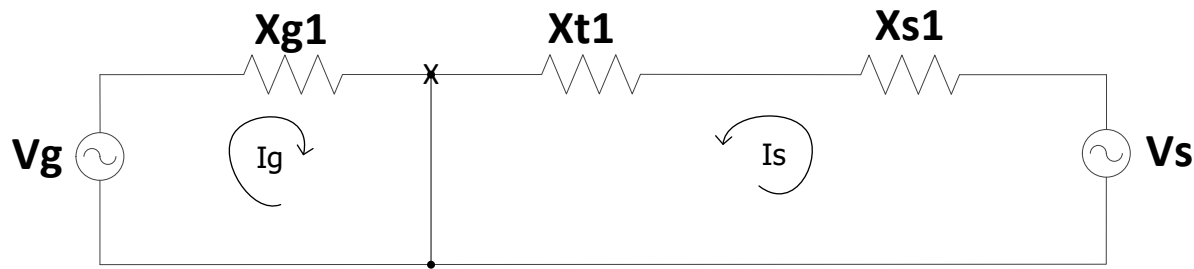
$$I_{sact} = I_{spu} * I_{baseLS} = 4.405 * \frac{700 * 1000}{\sqrt{3} * 19} = 93,698 \text{ pri amps on LS of GSU}$$

$$I_{sact} = I_{spu} * I_{baseHS} = 4.405 * \frac{700 * 1000}{\sqrt{3} * 345} = 5160 \text{ pri amps as seen from HS}$$

Fault Calculation Examples

3 phase fault on low side of GSU – Subtransient

- Alternate method which may only be used for 3 phase faults:



$$I_d'' = I_{gpu}'' * I_{baseLS} = \frac{1}{X_{g1}} * I_{baseLS} = \frac{1}{0.224} * \frac{700 * 1000}{\sqrt{3} * 19} = 94,959 \text{ pri amps}$$

$$I_{sact} = I_{spu} * I_{baseLS} = \frac{1}{X_{t1} + X_{s1}} * I_{baseLS} = \frac{1}{0.123 + 0.104} * \frac{700 * 1000}{\sqrt{3} * 19} \\ = 93,704 \text{ pri amps}$$

Fault Calculation Examples

3 phase fault on low side of GSU – Transient

- Calculate fault current using the generator transient reactance (X_d'):

$$I_{d'} = I_{gact'} = I_{gpu'} * I_{baseLS} = \frac{1}{X_{g1}} * I_{baseLS} = \frac{1}{0.295} * \frac{700 * 1000}{\sqrt{3} * 19}$$

$= 72,104 \text{ pri amps}$

- The system contribution will be the same as previously calculated.

Fault Calculation Examples

3 phase fault on low side of GSU – Synchronous

- Calculate fault current using the generator synchronous reactance (X_d):

$$I_d = I_{gact} = I_{gpu} * I_{baseLS} = \frac{1}{X_{g1}} * I_{baseLS} = \frac{1}{1.66} * \frac{700 * 1000}{\sqrt{3} * 19}$$
$$= 12,814 \text{ pri amps}$$

- Compare the synchronous fault current to the generator rated current:

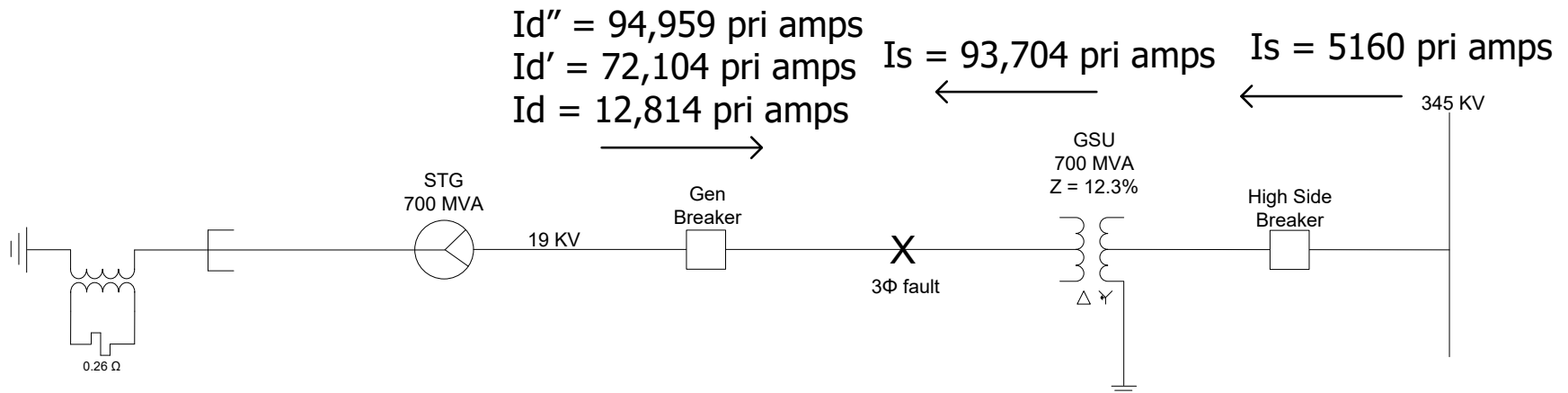
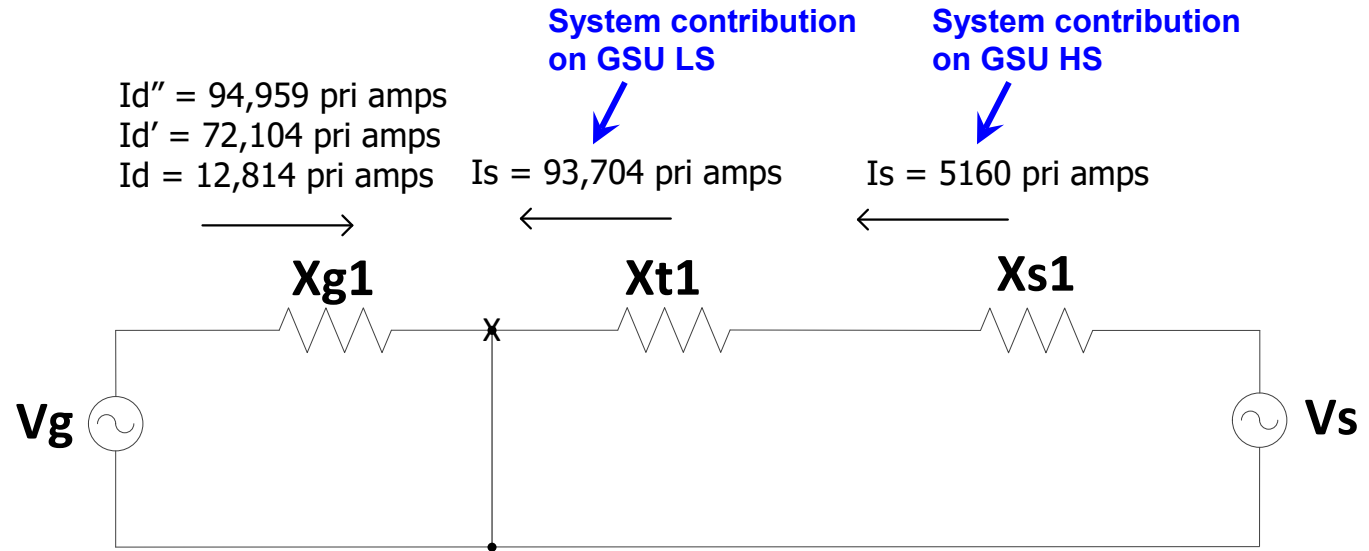
$$I_{rated} = \frac{S_{rated}}{\sqrt{3} * V_{LL}} = \frac{700 * 1000}{\sqrt{3} * 19} = 21,271 \text{ pri amps}$$

- That is why cannot use 51 function for system backup protection. Must use 51V instead i.e. because $I_d < I_{rated}$, use 51V function rather than 51 function (or use 21 function).
- The system contribution will be the same.

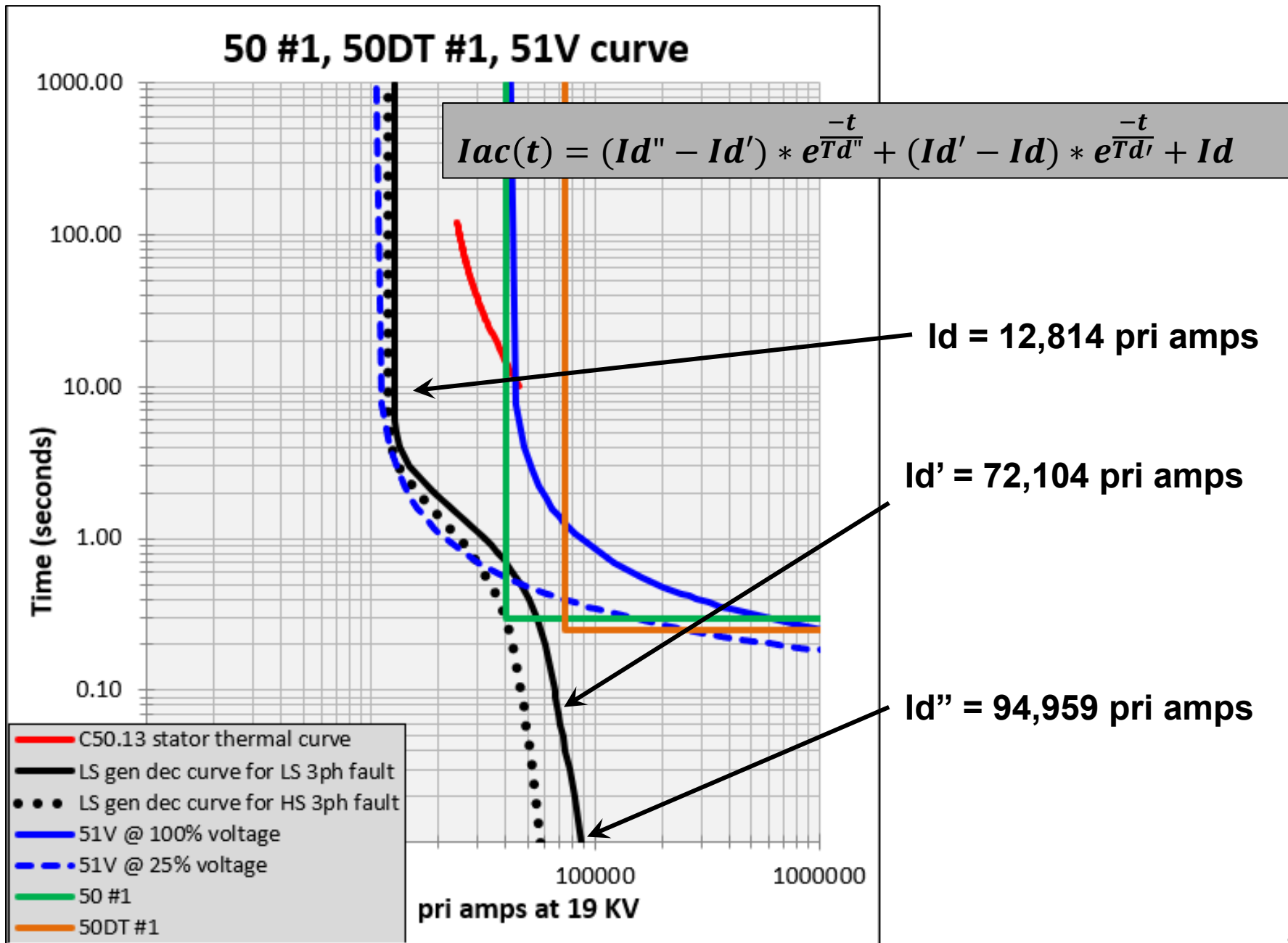
Fault Calculation Examples

3 phase fault on low side of GSU – Summary

- Subtransient, transient, synchronous gen and system contributions:

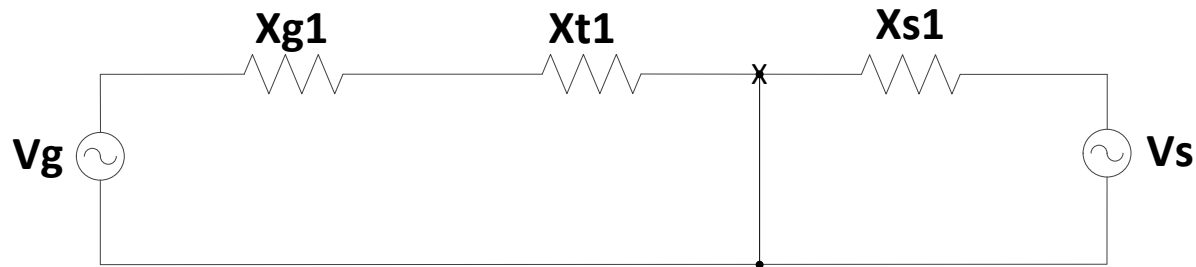
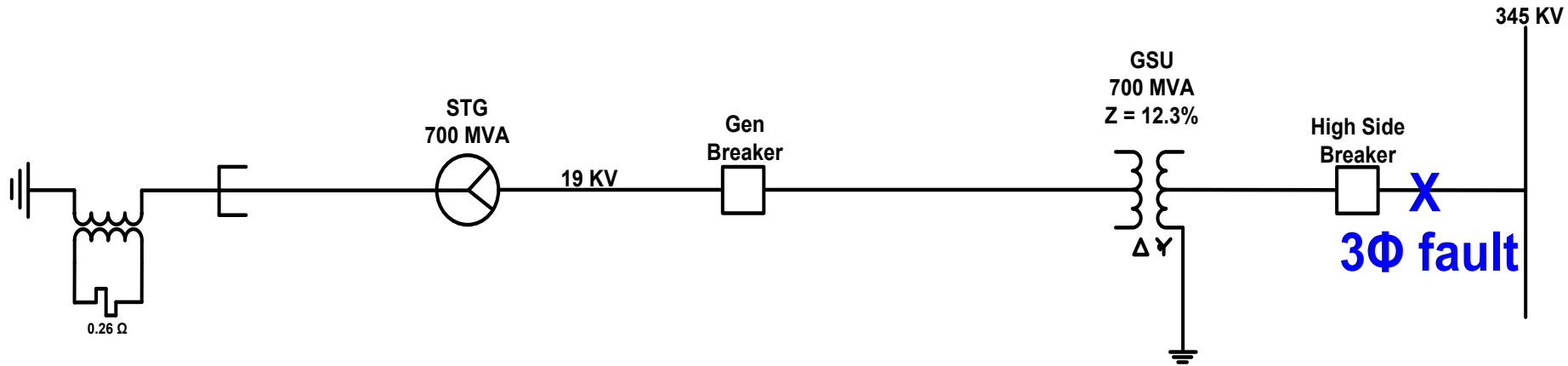


Fault Calculation Examples



Fault Calculation Examples

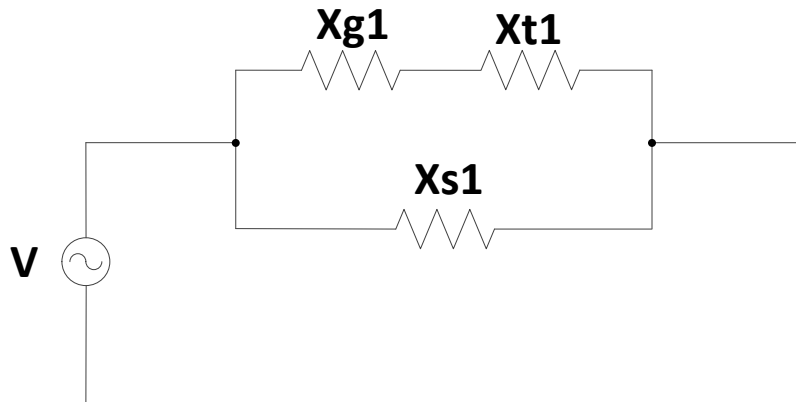
Case 2) 3 Φ fault on HS of GSU



Fault Calculation Examples

3 phase fault on high side of GSU – Subtransient

- Calculate the subtransient fault current contribution (use X_d'' for X_{g1}).
- Assume system voltage is at nominal value prior to the fault i.e. 1 pu.
- Assume all sources are in phase and of equal magnitude, which is equivalent to neglecting prefault load current.
- Therefore $V = V_g = V_s = 1$ pu at 0° , and network reduces as such:

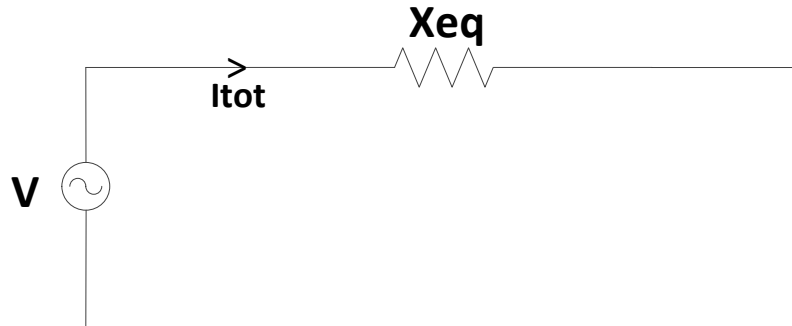


Fault Calculation Examples

3 phase fault on high side of GSU – Subtransient

- Calculate X_{eq} (using X_d'' for X_{g1}):

$$X_{eq} = \frac{(X_{g1} + X_{t1}) * X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = \frac{(0.224 + 0.123) * 0.104}{0.224 + 0.123 + 0.104} = 0.08 pu$$



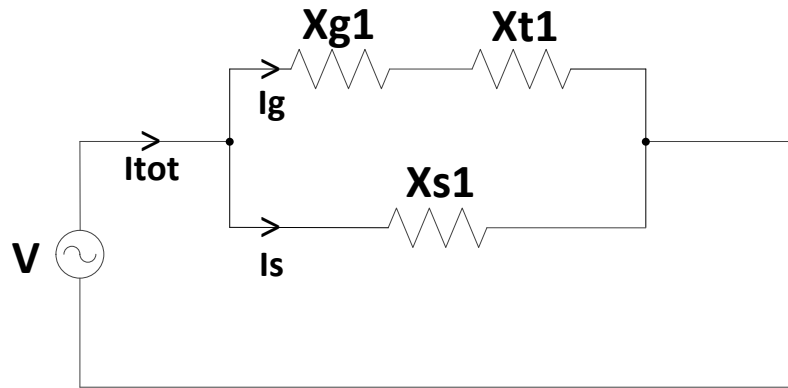
- Calculate I_{tot} :

$$I_{tot} = \frac{V}{X_{eq}} = \frac{1}{0.08} = 12.5 pu$$

Fault Calculation Examples

3 phase fault on high side of GSU – Subtransient

- Use current division to calculate the gen and system contributions:



$$I_{gpu}'' = I_{tot} * \frac{X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = 12.5 * \frac{0.104}{0.224 + 0.123 + 0.104} = 2.8825 pu$$

$$I_{spu} = I_{tot} * \frac{X_{g1} + X_{t1}}{X_{g1} + X_{t1} + X_{s1}} = 12.5 * \frac{0.224 + 0.123}{0.224 + 0.123 + 0.104} = 9.6175 pu$$

Fault Calculation Examples

3 phase fault on high side of GSU – Subtransient

- Convert from per unit to actual amps:

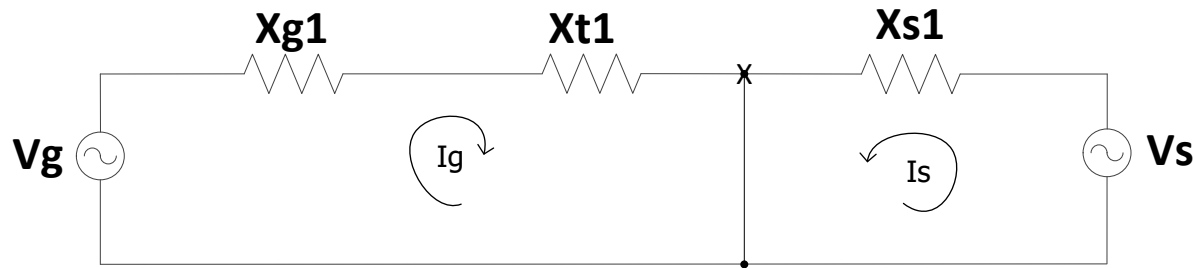
$$I_d'' = I_{gact}'' = I_{gpu}'' * I_{baseHS} = 2.8825 * \frac{700 * 1000}{\sqrt{3} * 345} = 3377 \text{ pri amps}$$

$$I_{sact} = I_{spu} * I_{baseHS} = 9.6175 * \frac{700 * 1000}{\sqrt{3} * 345} = 11,266 \text{ pri amps}$$

Fault Calculation Examples

3 phase fault on high side of GSU – Subtransient

- Alternate method for 3 phase faults only:



$$I_d'' = I_{gact}'' = I_{gpu}'' * I_{baseHS} = \frac{1}{X_{g1} + X_{t1}} * I_{baseHS}$$

$$= \frac{1}{0.224 + 0.123} * \frac{700 * 1000}{\sqrt{3} * 345} = 3376 \text{ pri amps}$$

$$\text{as seen on low side of GSU} = I_d'' * \frac{V_{baseHS}}{V_{baseLS}} = 3376 * \frac{345}{19} = 61,299 \text{ pri amps}$$

$$I_{sact} = I_{spu} * I_{baseHS} = \frac{1}{X_{s1}} * I_{baseHS} = \frac{1}{0.104} * \frac{700 * 1000}{\sqrt{3} * 345}$$
$$= 11,264 \text{ pri amps}$$

Fault Calculation Examples

3 phase fault on high side of GSU – Transient

- Calculate fault current using the generator transient reactance (X_d'):

$$I_{d'} = I_{gact'} = I_{gpu'} * I_{baseHS} = \frac{1}{X_{g1} + X_{t1}} * I_{baseHS}$$
$$= \frac{1}{0.295 + 0.123} * \frac{700 * 1000}{\sqrt{3} * 345} = 2802 \text{ pri amps}$$

$$\text{as seen on low side of GSU} = I_{d'} * \frac{V_{baseHS}}{V_{baseLS}} = 2802 * \frac{345}{19} = 50,887 \text{ pri amps}$$

- The system contribution will be the same.

Fault Calculation Examples

3 phase fault on high side of GSU – Synchronous

- Calculate fault current using the generator synchronous reactance (X_d):

$$I_d = I_{gact} = I_{gpu} * I_{baseHS} = \frac{1}{X_{g1} + X_{t1}} * I_{baseHS} = \frac{1}{1.66 + 0.123} * \frac{700 * 1000}{\sqrt{3} * 345}$$

$= 657 \text{ pri amps}$

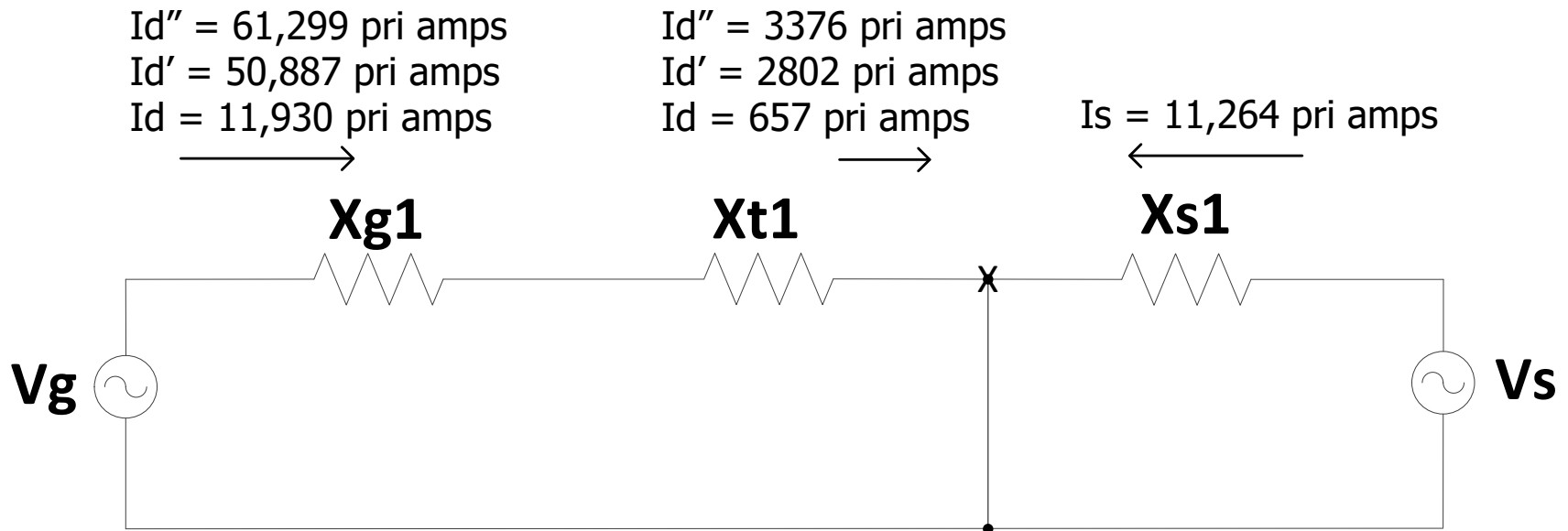
$$\text{as seen on low side of GSU} = I_d * \frac{V_{baseHS}}{V_{baseLS}} = 657 * \frac{345}{19} = 11,930 \text{ pri amps}$$

- The system contribution will be the same.

Fault Calculation Examples

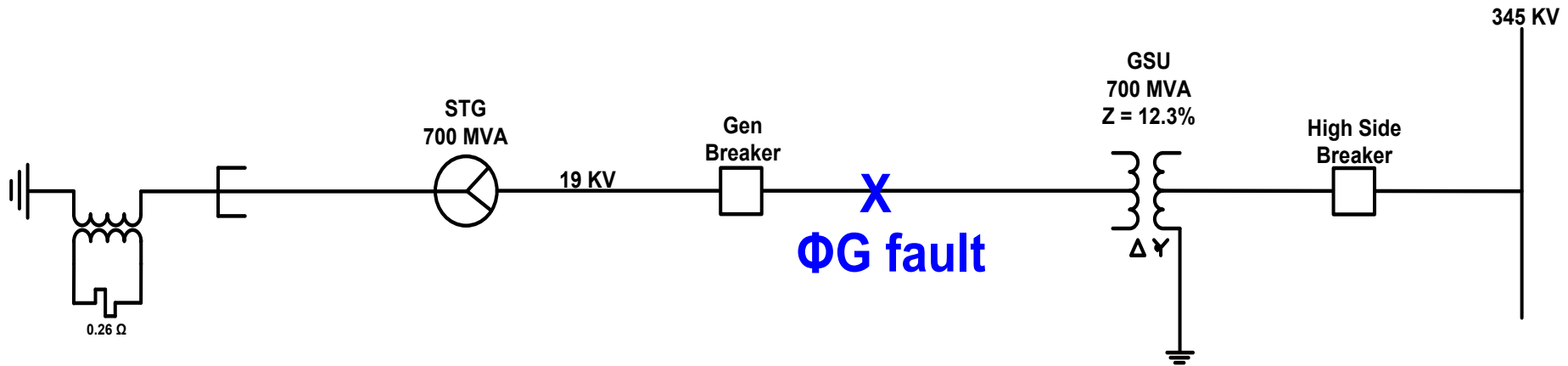
3 phase fault on high side of GSU – Summary

- Subtransient, transient, and synchronous generator contributions to the high side fault showing fault amps on HS and as seen from the LS of the GSU, along with the system contribution to the fault:



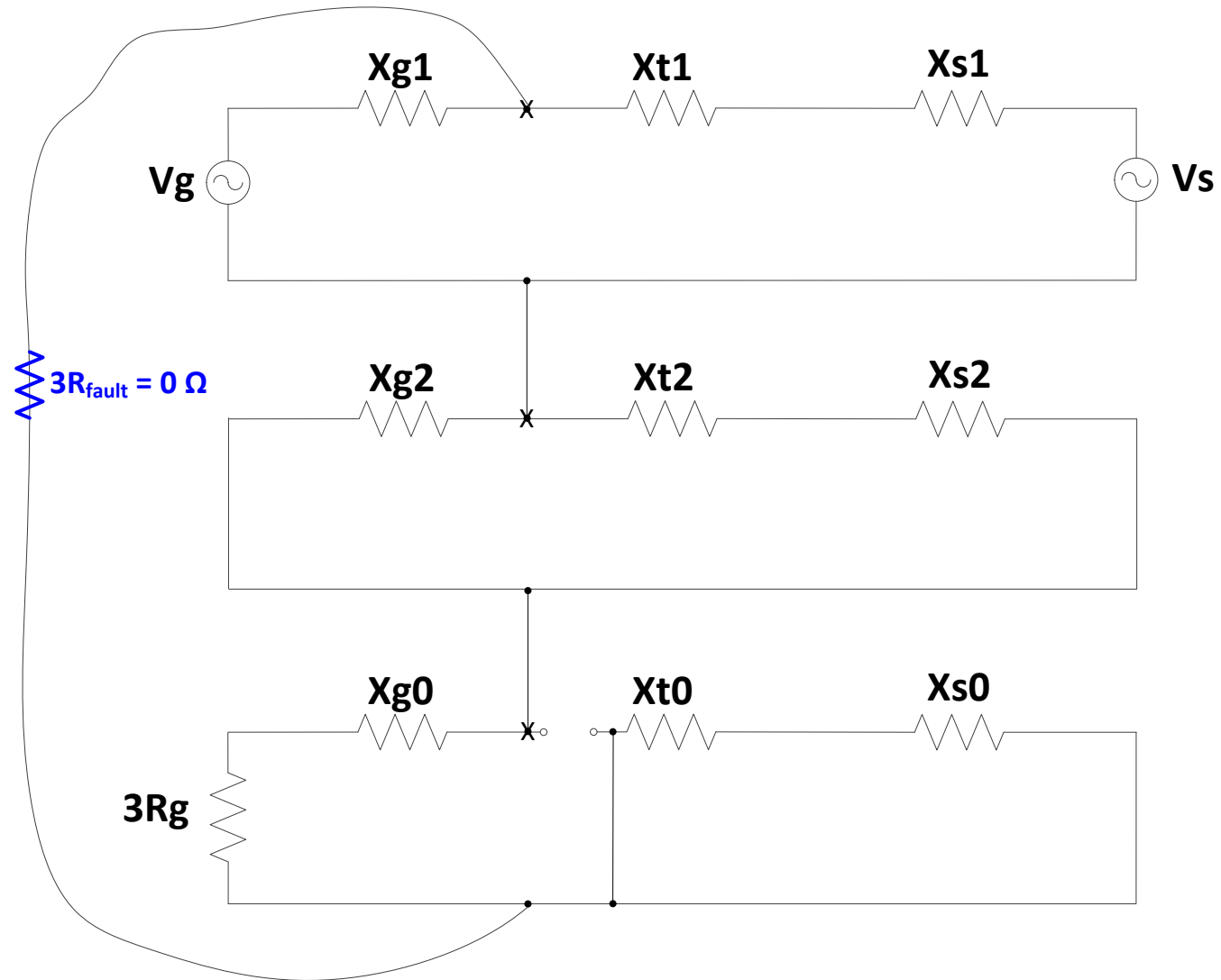
Fault Calculation Examples

Case 3) Φ G fault on LS of GSU



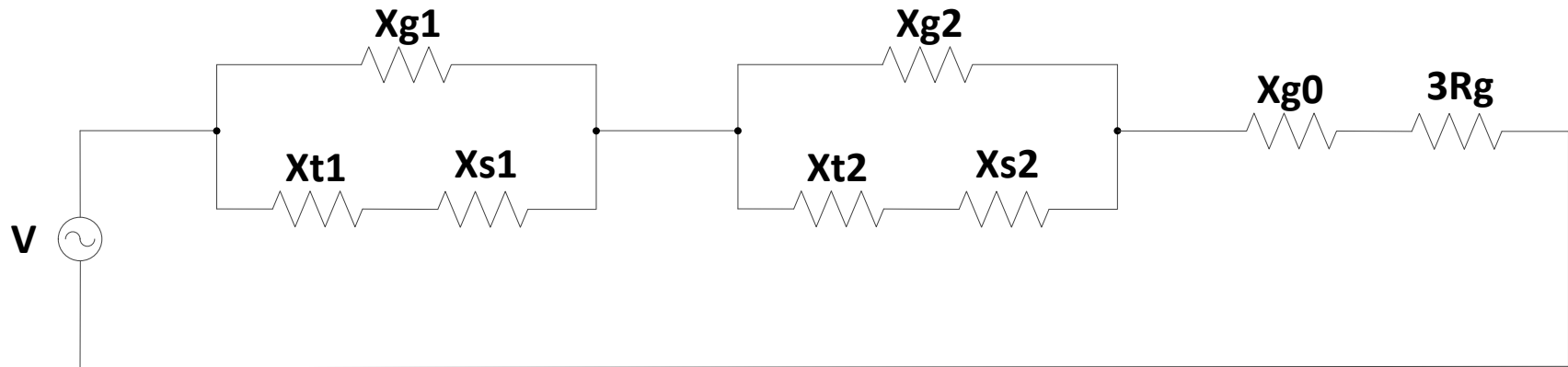
Fault Calculation Examples

Phase-Ground fault on LS of GSU



Fault Calculation Examples

Phase-Ground fault on LS of GSU – Subtransient



- Calculate equivalent reactances in each network (use X_d'' for X_{g1}):

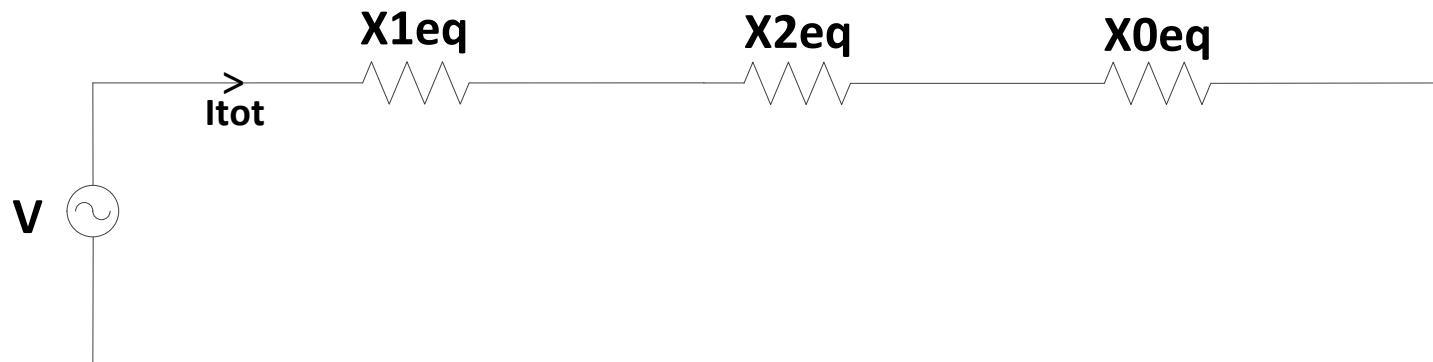
$$X_{1eq} = \frac{X_{g1} * (X_{t1} + X_{s1})}{X_{g1} + X_{t1} + X_{s1}} = \frac{0.224 * (0.123 + 0.104)}{0.224 + 0.123 + 0.104} = 0.113 \text{ pu}$$

$$X_{2eq} = \frac{X_{g2} * (X_{t2} + X_{s2})}{X_{g2} + X_{t2} + X_{s2}} = \frac{0.224 * (0.123 + 0.104)}{0.224 + 0.123 + 0.104} = 0.113 \text{ pu}$$

$$X_{0eq} = X_{g0} + 3R_g = 0.128 + 3 * 4206.7 = 12,620 \text{ pu}$$

Fault Calculation Examples

Phase-Ground fault on LS of GSU – Subtransient



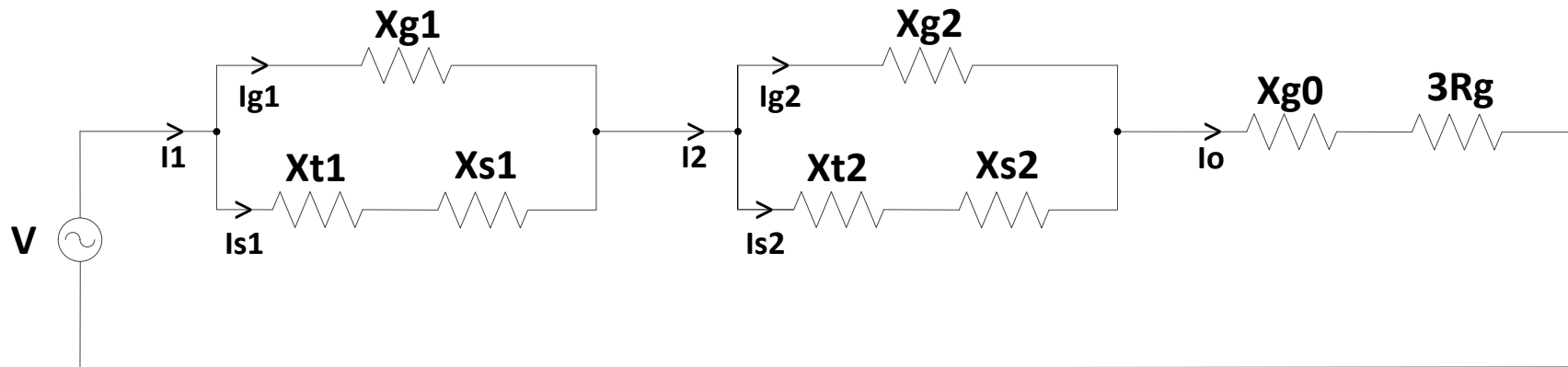
$$I_{tot} = \frac{V}{X_{1eq} + X_{2eq} + X_{0eq}} = \frac{1}{0.113 + 0.113 + 12,620} = 0.00008 \text{ pu}$$

$$I_{tot} = I_1 = I_2 = I_o$$

Fault Calculation Examples

Phase-Ground fault on LS of GSU – Subtransient

- Break apart the equivalent reactances and use current division to calculate the gen and system contributions to the fault:



Fault Calculation Examples

Phase-Ground fault on LS of GSU – Subtransient

$$I_{g1} = I_1 * \frac{X_{t1} + X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = 0.00008 * \frac{0.123 + 0.104}{0.224 + 0.123 + 0.104} = 0.00004 \text{ pu}$$

$$I_{s1} = I_1 - I_{g1} = 0.00008 - 0.00004$$

$$I_{s1} = 0.00004 \text{ pu}$$

$$I_{g2} = I_2 * \frac{X_{t2} + X_{s2}}{X_{g2} + X_{t2} + X_{s2}} = 0.00008 * \frac{0.123 + 0.104}{0.224 + 0.123 + 0.104} = 0.00004 \text{ pu}$$

$$I_{s2} = I_2 - I_{g2} = 0.00008 - 0.00004 = 0.00004 \text{ pu}$$

$$I_{s2} = 0.00004 \text{ pu}$$

$$I_{g0} = I_o = 0.00008 \text{ pu}$$

$$I_{s0} = 0$$

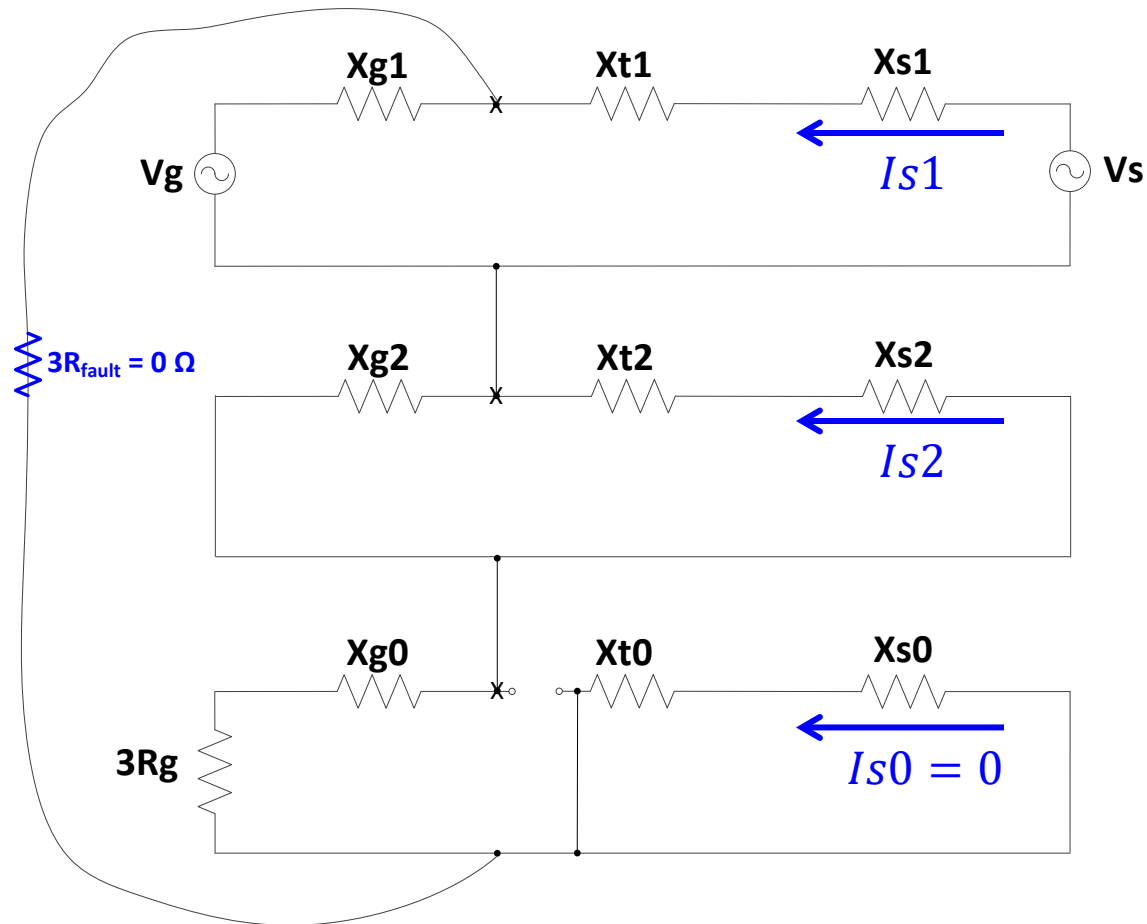
- QUESTION: Notice that the system contributes positive and negative sequence current to the fault, but no zero-sequence current.

- Why is that?

Fault Calculation Examples

Phase-Ground fault on LS of GSU

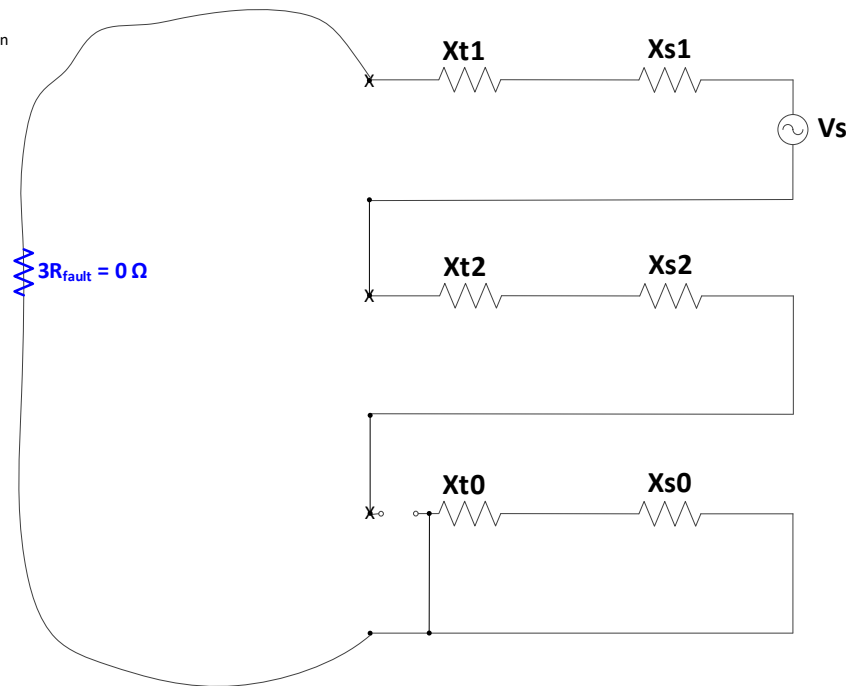
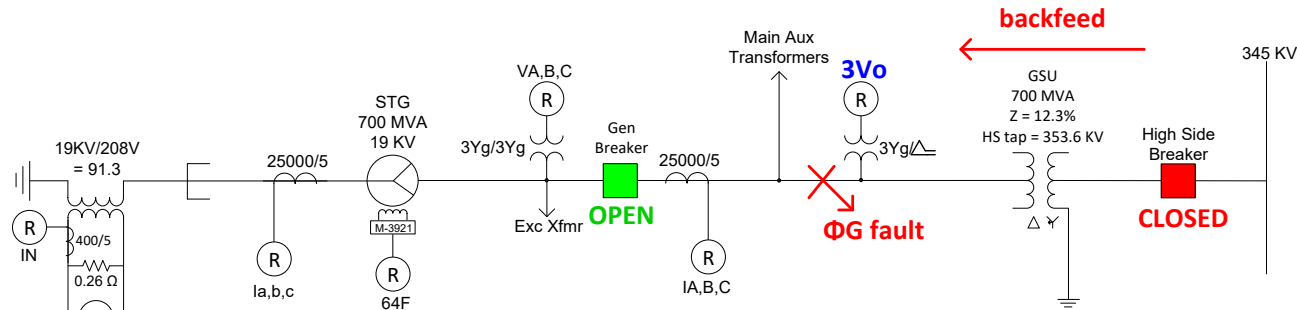
- ANSWER: Because of the GSU connection (Delta-LS/grounded Wye-HS)



Fault Calculation Examples

Phase-Ground fault on LS of GSU

- How about if the generator is off-line (LS gen breaker is open) and the GSU HS breaker is closed while a ΦG fault occurs on the GSU LS?



There is no continuous circuit so no 3I₀ fault current will flow.

Typically, a calculated 3V₀ from (3) Yg/Yg VTs or a measured 3V₀ from a broken delta VTs is used for bus ground fault protection when the gen is off-line and the GSU is back-feeding local plant aux load.

Fault Calculation Examples

Phase-Ground fault on LS of GSU – Subtransient

- Convert from per unit to actual amps:

$$I_s = (I_{s1} + I_{s2} + I_{s0}) * I_{baseLS} = (0.00004 + 0.00004 + 0) * \frac{700 * 1000}{\sqrt{3} * 19}$$

$= 1.7 \text{ pri amps}$ ($1.7 * 19 / 345 = 0.09 \text{ A}$ as seen on HS)

- System contribution is still so small, why? Because of NGR

$$I_g = (I_{g1} + I_{g2} + I_{g0}) * I_{baseLS} = (0.00004 + 0.00004 + 0.00008) * \frac{700 * 1000}{\sqrt{3} * 19}$$

$= 3.4 \text{ pri amps}$

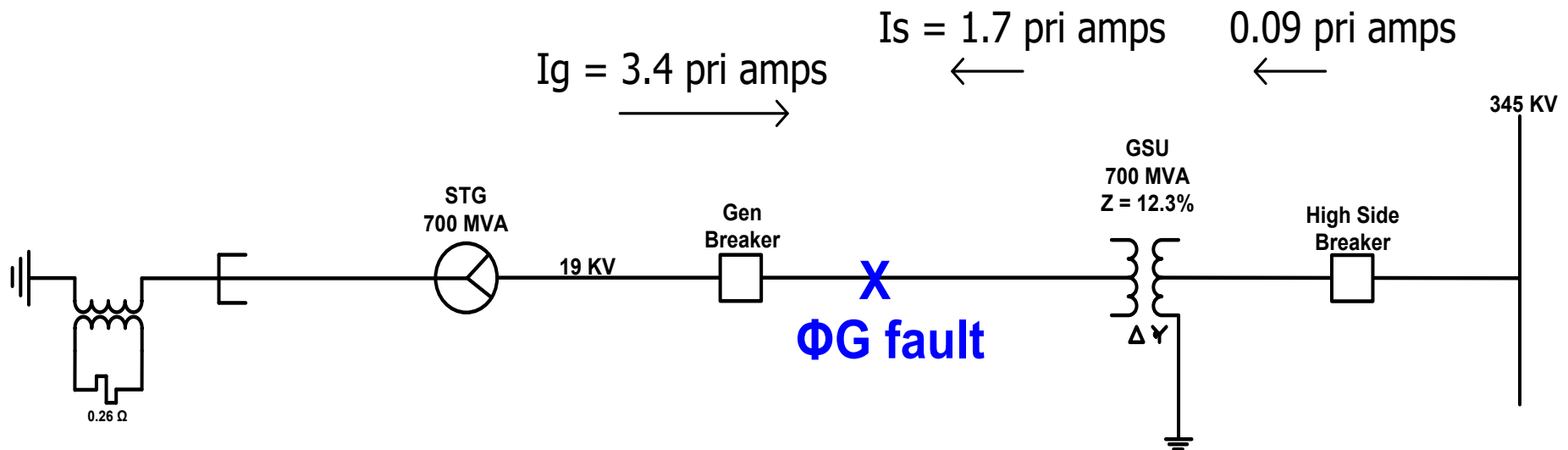
$$I_{tot} = I_s + I_g = 1.7 + 3.4 = 5.1 \text{ pri amps}$$

$$\text{check: } 3I_o = 3 * I_o * I_{baseLS} = 3 * 0.00008 * \frac{700 * 1000}{\sqrt{3} * 19} = 5.1 \text{ pri amps}$$

- Transient and Synchronous fault current calculations come out similar.

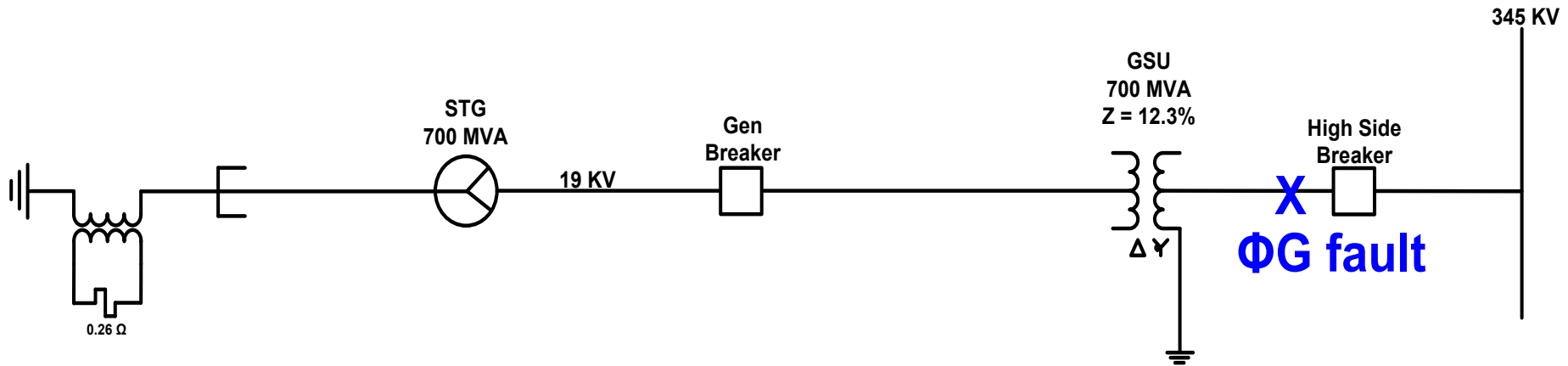
Fault Calculation Examples

Phase-Ground fault on LS of GSU – Summary



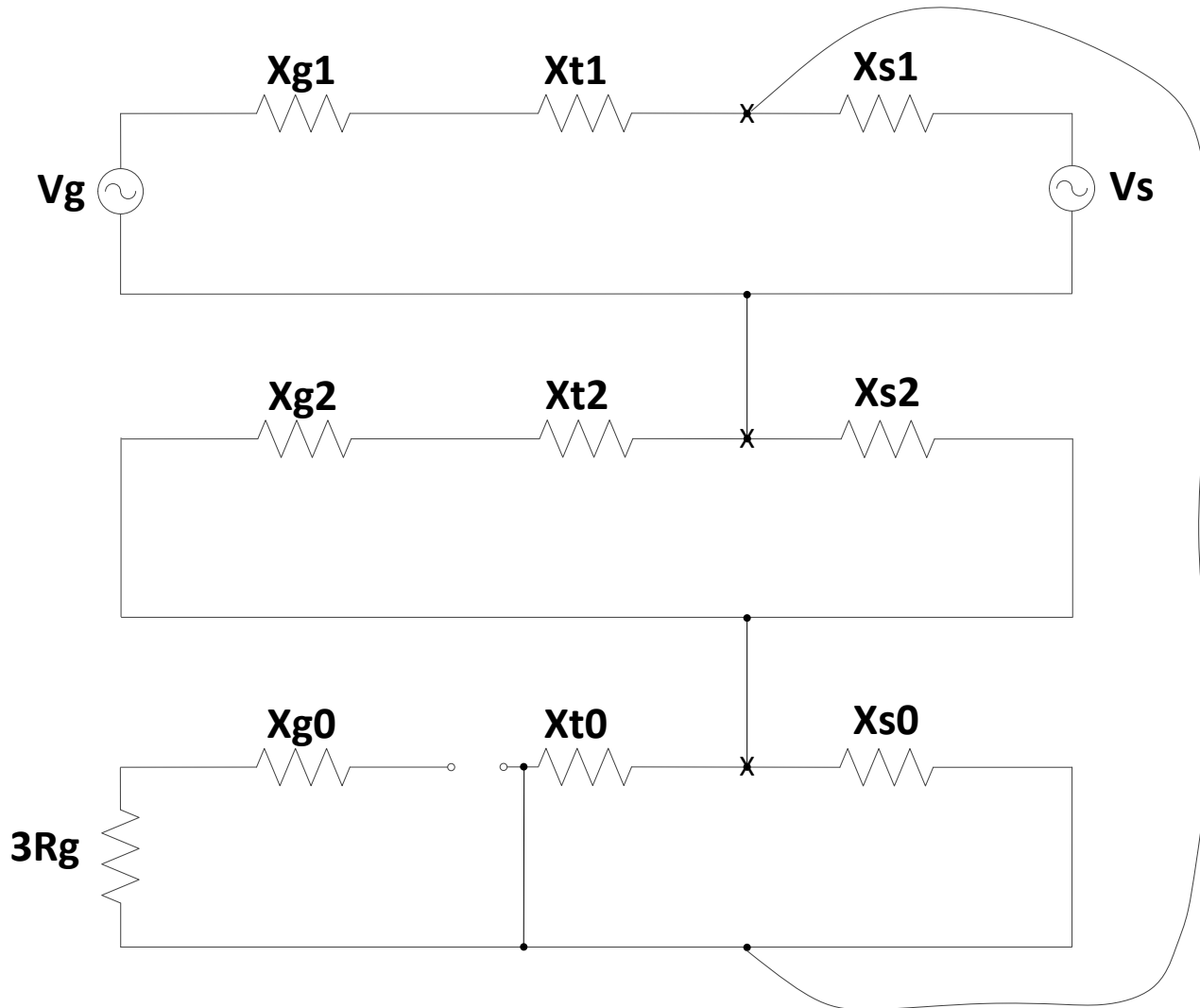
Fault Calculation Examples

Case 4) Φ G fault on HS of GSU



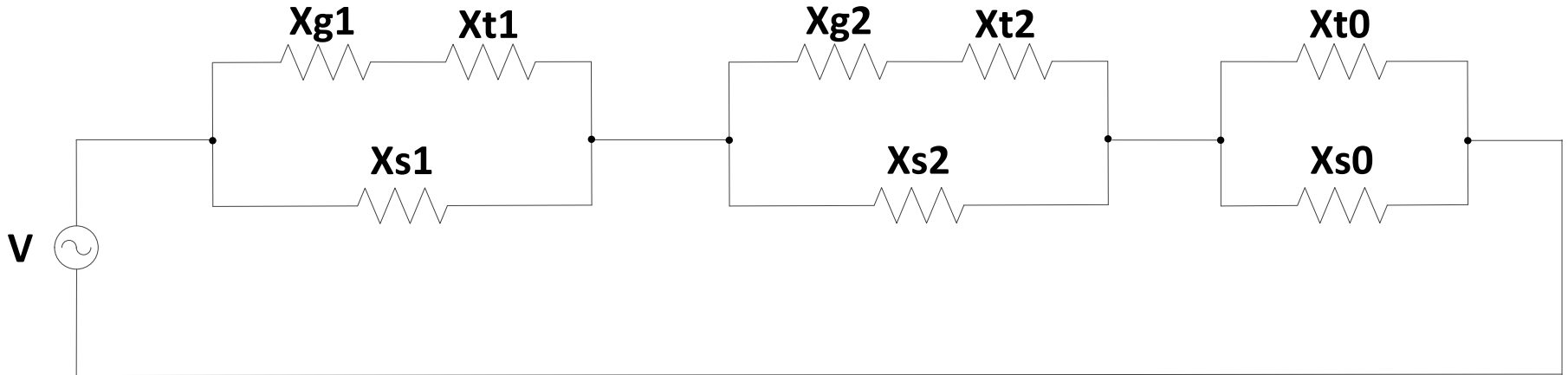
Fault Calculation Examples

Phase-Ground fault on HS of GSU



Fault Calculation Examples

Phase-Ground fault on HS of GSU – Subtransient



- Calculate equivalent reactances in each network (using X_d'' for X_{g1}):

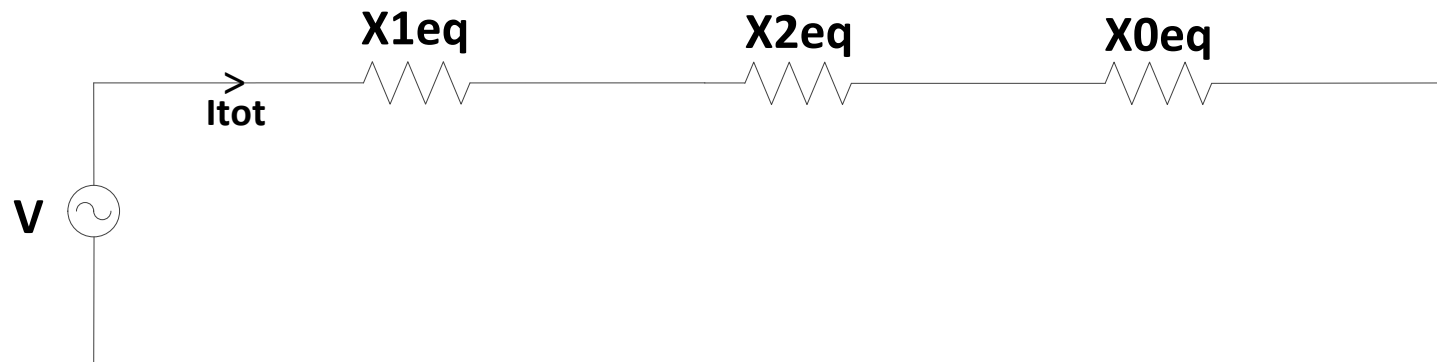
$$X_{1eq} = \frac{(X_{g1} + X_{t1}) * X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = \frac{(0.224 + 0.123) * 0.104}{0.224 + 0.123 + 0.104} = 0.08 \text{ pu}$$

$$X_{2eq} = \frac{(X_{g2} + X_{t2}) * X_{s2}}{X_{g2} + X_{t2} + X_{s2}} = \frac{(0.224 + 0.123) * 0.104}{0.224 + 0.123 + 0.104} = 0.08 \text{ pu}$$

$$X_{0eq} = \frac{X_{t0} * X_{s0}}{X_{t0} + X_{s0}} = \frac{0.123 * 0.093}{0.123 + 0.093} = 0.053 \text{ pu}$$

Fault Calculation Examples

Phase-Ground fault on HS of GSU – Subtransient



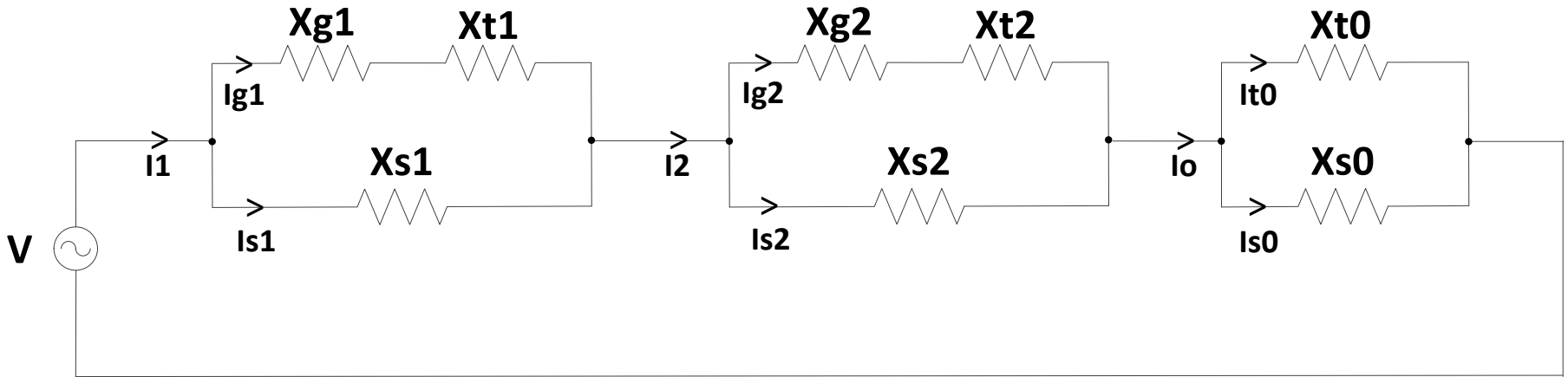
$$I_{tot} = \frac{V}{X_{1eq} + X_{2eq} + X_{0eq}} = \frac{1}{0.08 + 0.08 + 0.053} = 4.7 \text{ pu}$$

$$I_{tot} = I_1 = I_2 = I_0$$

Fault Calculation Examples

Phase-Ground fault on HS of GSU – Subtransient

- Break up the equivalent reactances and use current division to calculate the gen and system contributions to the fault.



Fault Calculation Examples

Phase-Ground fault on HS of GSU – Subtransient

$$I_{g1} = I_1 * \frac{X_{s1}}{X_{g1} + X_{t1} + X_{s1}} = 4.7 * \frac{0.104}{0.224 + 0.123 + 0.104} = 1.08 \text{ pu}$$

$$I_{s1} = I_1 - I_{g1} = 4.7 - 1.08 = 3.62 \text{ pu}$$

$$I_{g2} = I_2 * \frac{X_{s2}}{X_{g2} + X_{t2} + X_{s2}} = 4.7 * \frac{0.104}{0.224 + 0.123 + 0.104} = 1.08 \text{ pu}$$

$$I_{s2} = I_2 - I_{g2} = 4.7 - 1.08 = 3.62 \text{ pu}$$

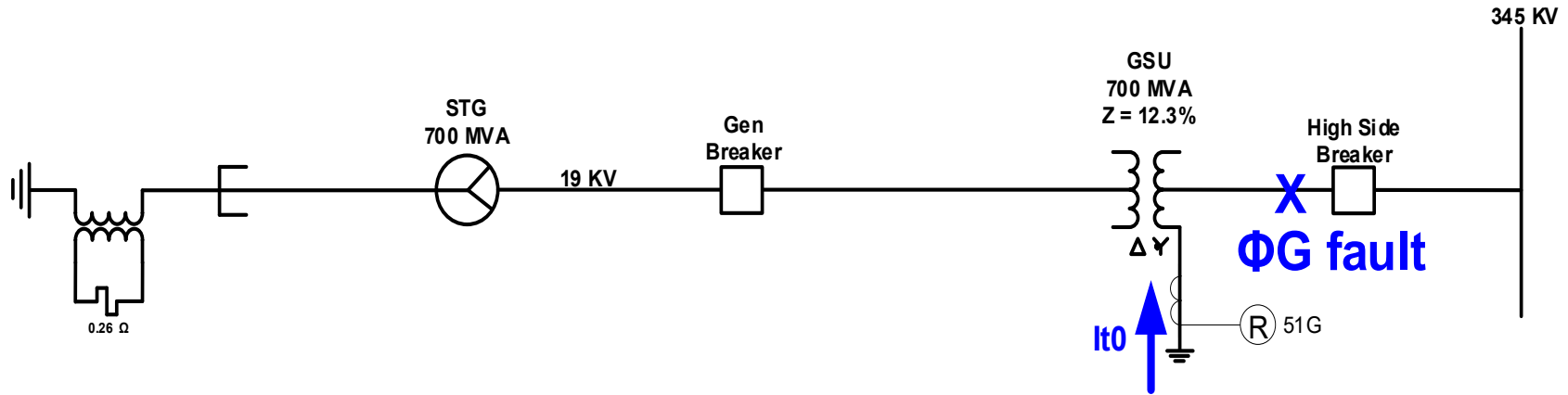
$$I_{t0} = I_o * \frac{X_{s0}}{X_{t0} + X_{s0}} = 4.7 * \frac{0.093}{0.123 + 0.093} = 2 \text{ pu}$$

$$I_{s0} = I_o - I_{t0} = 4.7 - 2 = 2.7 \text{ pu}$$

- Generator does not contribute any zero-sequence current to the fault, only positive and negative sequence current.
- However, $3I_o$ current does return up thru the GSU ground leg which allows use of a **51G relay as ground fault system backup protection.**

Fault Calculation Examples

Sidebar: Determine 51G relay Pickup



$$I_{t0} = I_{pu} * I_{base} = 2 * \frac{700 * 1000}{\sqrt{3} * 345} = 2343 \text{ pri amps}, \quad \frac{2343}{240} = 9.76 \text{ sec amps}$$

- Therefore, set 51G Pickup < 9.76 A with plenty sensitivity margin.
- For example, with a 25% sensitivity margin = 0.25*9.76 = 2.44 A.

3Io during max load (use 10% of max load) ≤ 51G Pickup ≤ min fault

$$0.10 * \frac{700 * 1000}{\sqrt{3} * 345} * \frac{5}{1200} \leq 51G \text{ Pickup} \leq 0.25 * 9.76$$

$$0.488 \leq 51G \text{ Pickup} \leq 2.44$$

Fault Calculation Examples

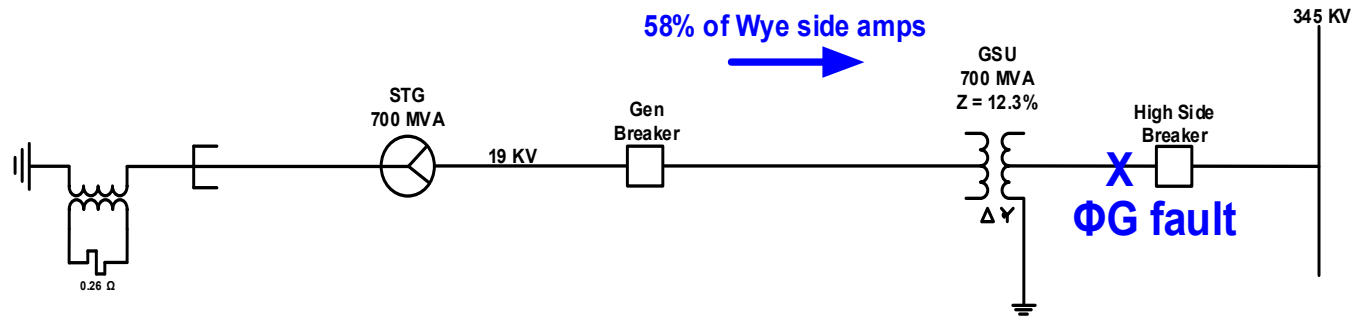
Phase-Ground fault on HS of GSU

$$I_s = (I_{s1} + I_{s2} + I_{s0}) * I_{baseHS} = (3.62 + 3.62 + 2.7) * \frac{700 * 1000}{\sqrt{3} * 345}$$
$$= 11,644 \text{ pri amps}$$

$$I_g = (I_{g1} + I_{g2} + I_{t0}) * I_{baseHS} = (1.08 + 1.08 + 2) * \frac{700 * 1000}{\sqrt{3} * 345} = 4873 \text{ pri amps}$$

$$\text{as seen on LS} = 0.577 * 4873 * \frac{345}{19} = 51,057 \text{ pri amps}$$

The Delta side will see 58% of the fault current for a ΦG fault on the Wye side.

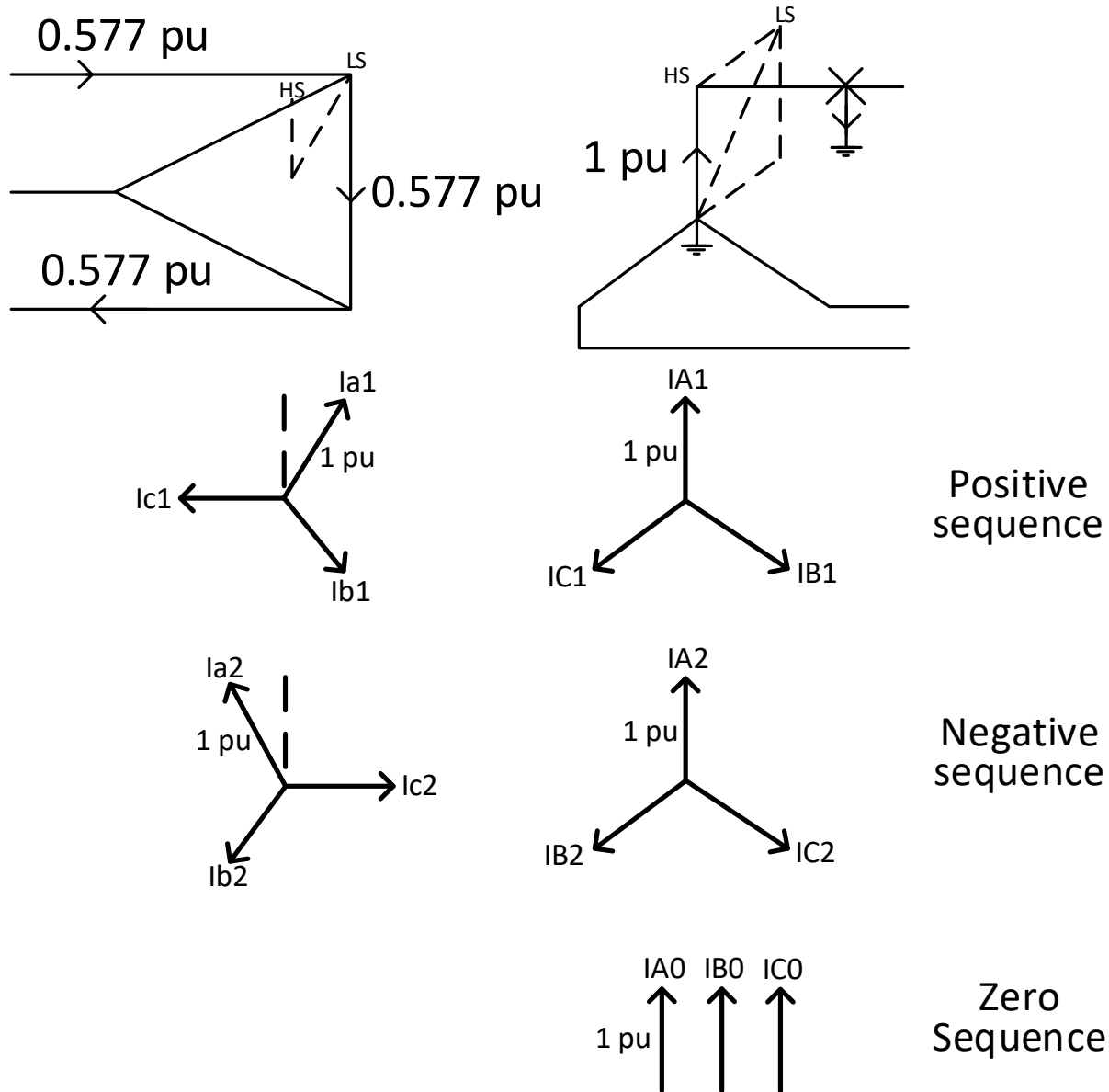


$$I_{tot} = I_s + I_g = 11,644 + 4873 = 16,517 \text{ pri amps}$$

$$\text{check: } 3I_o = 3 * I_o * I_{baseHS} = 3 * 4.7 * \frac{700 * 1000}{\sqrt{3} * 345} = 16,517 \text{ pri amps}$$

Fault Calculation Examples

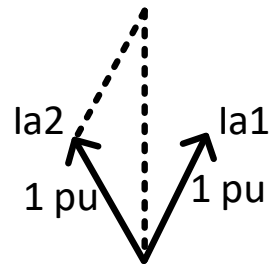
Sidebar: For a Wye-HS Φ G fault, the Delta-LS sees 58% of Wye-HS amps



Fault Calculation Examples

Sidebar: For a Wye-HS Φ G fault, the Delta-LS sees 58% of Wye-HS amps

Delta-LS

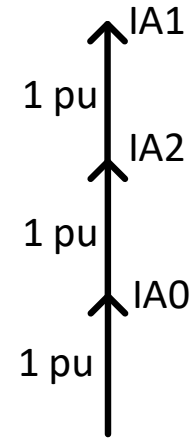


$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_a = 1pu @ 330^\circ + 1pu @ 30^\circ + 0$$

$$I_a = \sqrt{3}$$

Wye-HS



$$I_A = I_{A1} + I_{A2} + I_{A0}$$

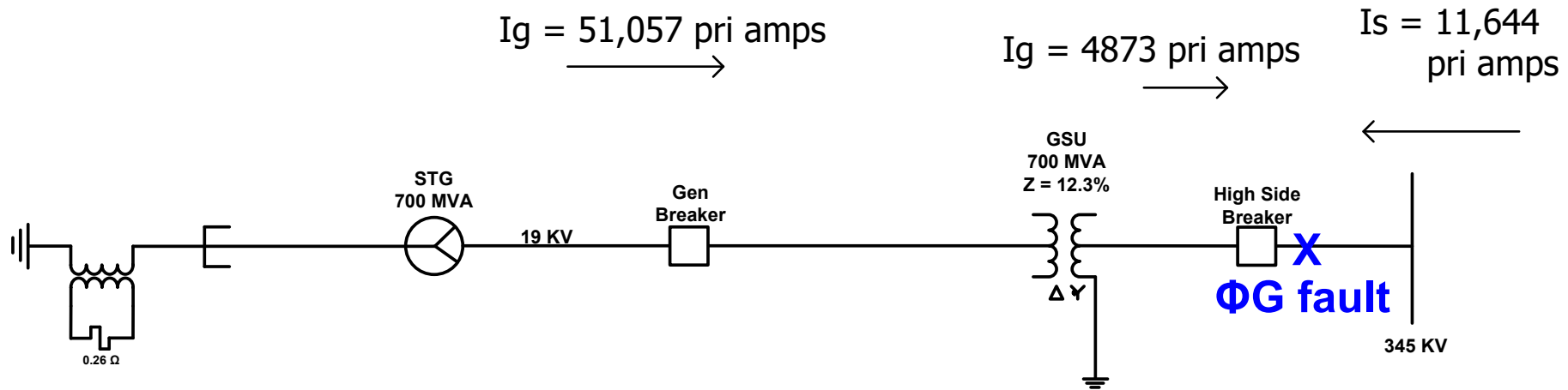
$$I_A = 1pu + 1pu + 1pu$$

$$I_A = 3$$

$$\frac{I_a}{I_A} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = 0.577$$

Fault Calculation Examples

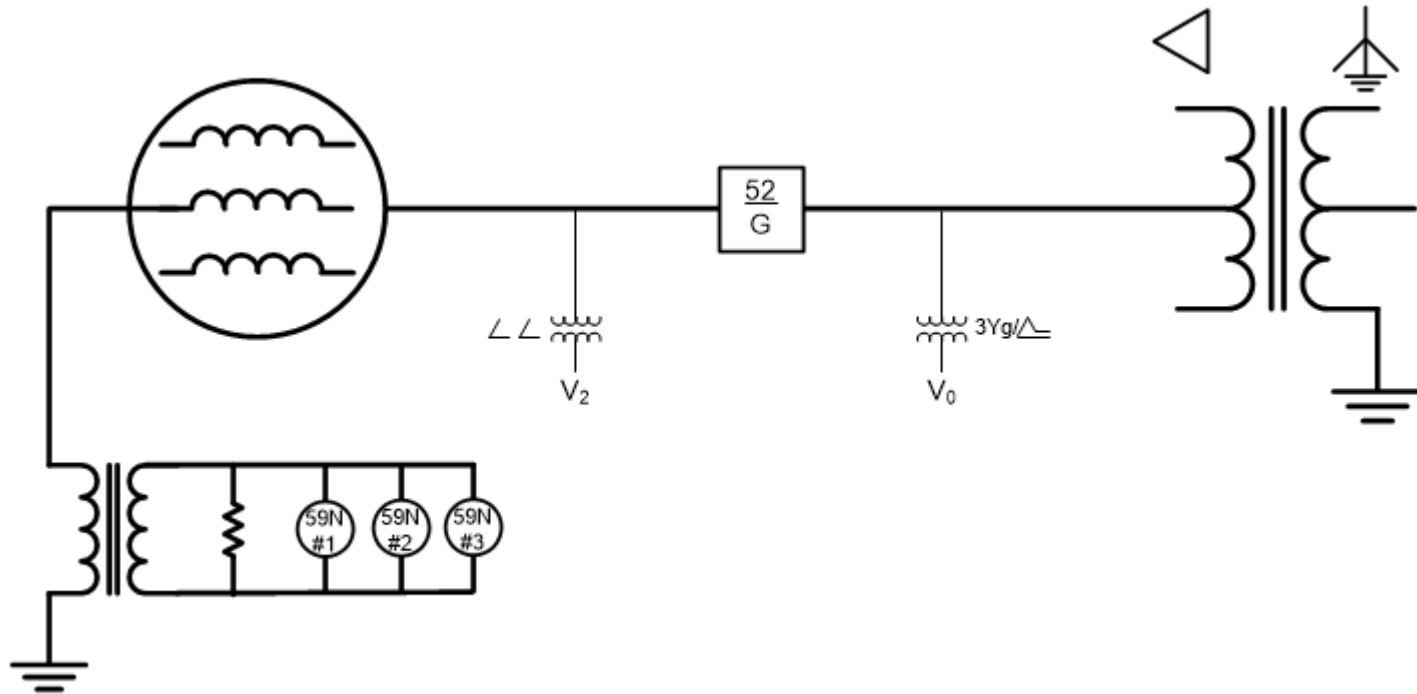
Phase-Ground fault on HS of GSU – Summary



- Because the generator does contribute negative sequence current to an unbalanced fault on the GSU HS, the 46 negative sequence overcurrent relay on the generator does need to coordinate with system relays for unbalanced faults.
- This is usually quite easy to coordinate as the 46 element in the generator relay typically has a much slower response time than that of system relays.

Fault Calculation Examples

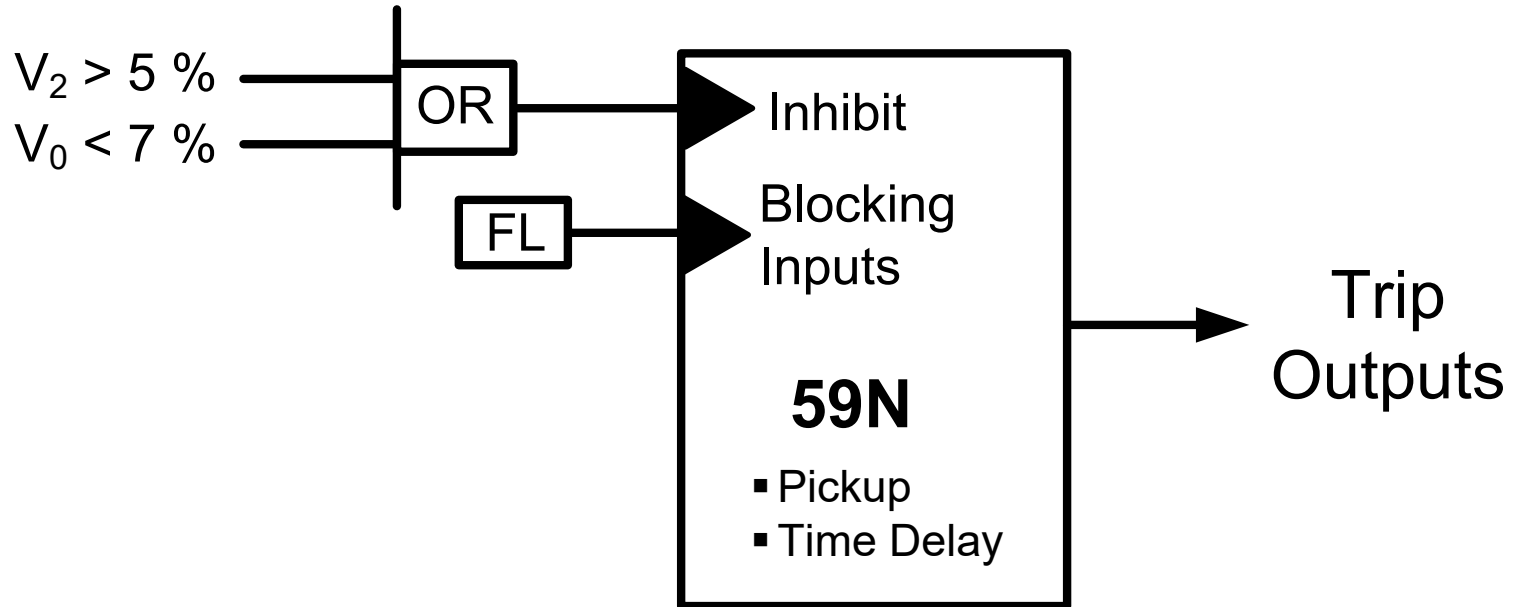
Case 5) V_2 and V_0 calcs for GSU HS and LS ΦG faults



59N – accelerated ground overvoltage scheme with V_2 and V_0 supervision

- $V_0 > V_2$ for GSU low side ground faults
- $V_0 < V_2$ for GSU high side ground faults

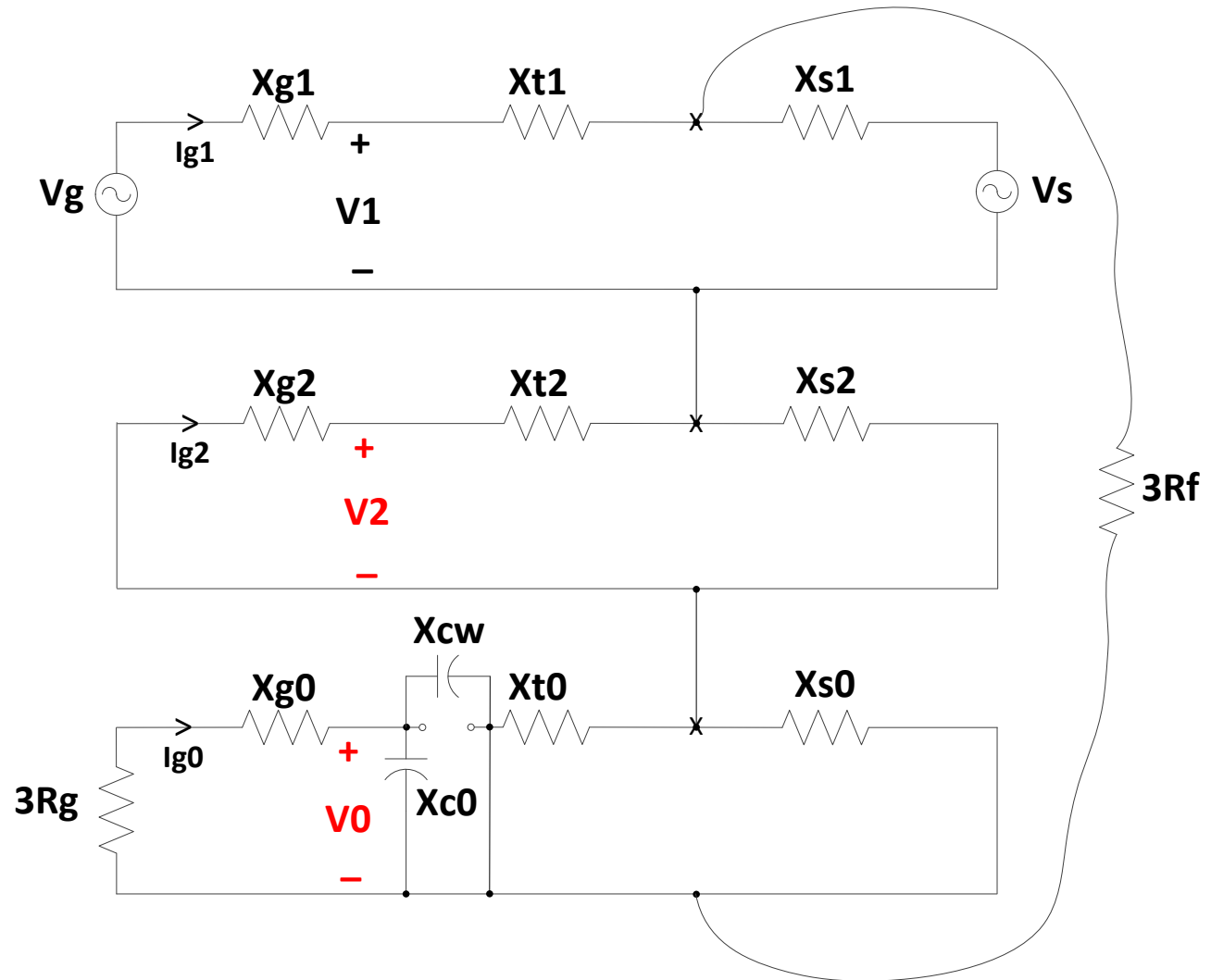
Fault Calculation Examples



Check to ensure that the chosen setpoints of 5% for V_2 and 7% for V_0 are OK for this application.

Fault Calculation Examples

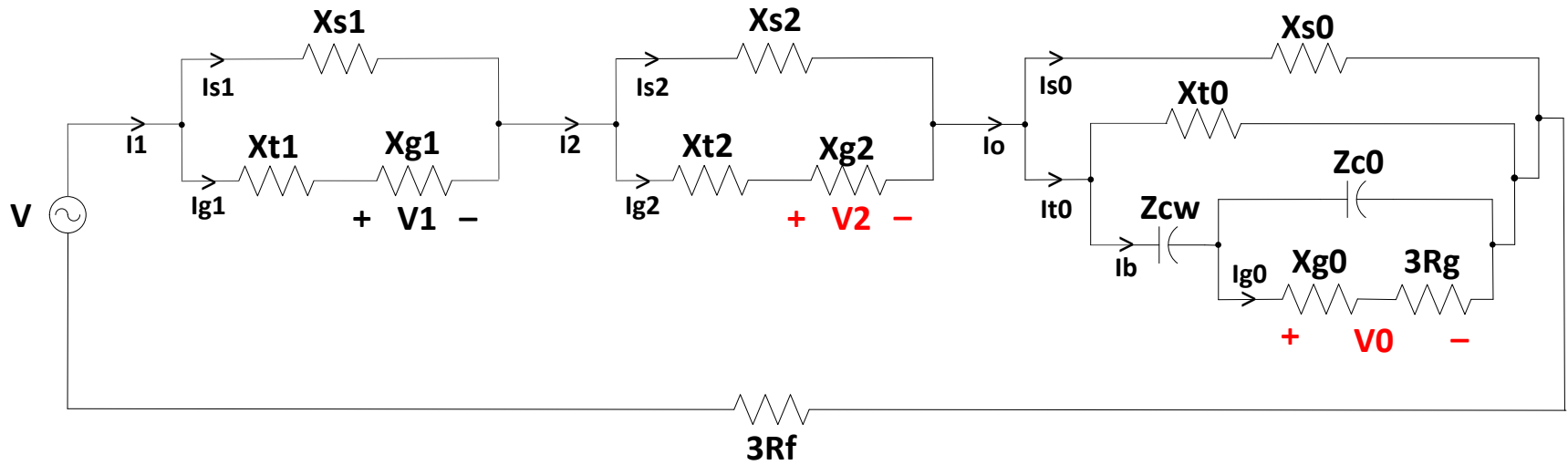
Φ G fault on HS of GSU – sequence network connections



- X_{cw} is the GSU interwinding capacitive reactance
- X_{c0} is the total capacitive reactance to ground i.e. stator winding, iso-phase bus, surge equip, etc

Fault Calculation Examples

Φ G fault on HS of GSU – network reduction



Fault Calculation Examples

ΦG fault on HS of GSU

$$Z1 := \frac{Zs1 \cdot (Zt + Xg1sub)}{Zs1 + Zt + Xg1sub} \quad Z2 := \frac{Zs2 \cdot (Zt + Xg2)}{Zs2 + Zt + Xg2} \quad Za := \frac{ZCo \cdot (Xg0 + 3 \cdot Rg)}{ZCo + Xg0 + 3 \cdot Rg} \quad Zb := Za + ZCw \quad Zc := \frac{Zt \cdot Zb}{Zt + Zb} \quad Z0 := \frac{Zs0 \cdot Zc}{Zs0 + Zc}$$

$$Io := \frac{1}{Z1 + Z2 + Z0 + 3 \cdot RfHS} \quad Ip := Io \quad In := Io$$

$$Igl := Ip \cdot \left(\frac{Zs1}{Zs1 + Zt + Xg1sub} \right) \quad Is1 := Ip - Igl$$

$$Ig2 := In \cdot \left(\frac{Zs2}{Zs2 + Zt + Xg2} \right) \quad Is2 := In - Ig2$$

$$It0 := Io \cdot \left(\frac{Zs0}{Zs0 + Zc} \right) \quad Is0 := Io - It0 \quad Ib := It0 \cdot \left(\frac{Zto}{Zto + Zb} \right) \quad Ig0 := Ib \cdot \left(\frac{ZCo}{ZCo + Xg0 + 3 \cdot Rg} \right)$$

$$V2 := Ig2 \cdot Xg2$$

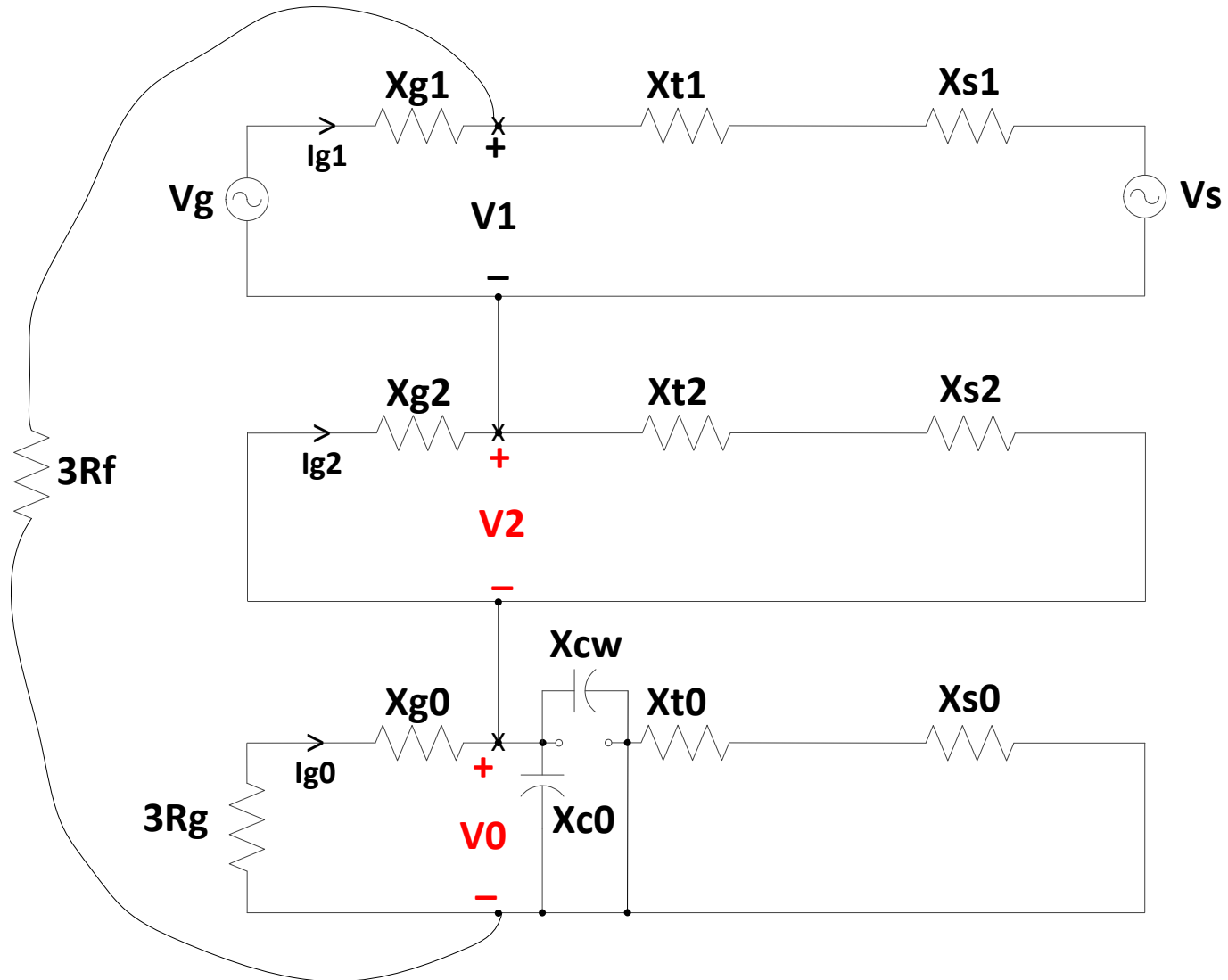
$$V0 := Ig0 \cdot Za$$

$|V2 \cdot 100| = 22$ %, and because $22\% > 5\%$, it will correctly block operation for this high side fault

$|V0 \cdot 100| = 0.19$ %, and because $0.19\% < 7\%$, it will correctly block operation for this high side fault

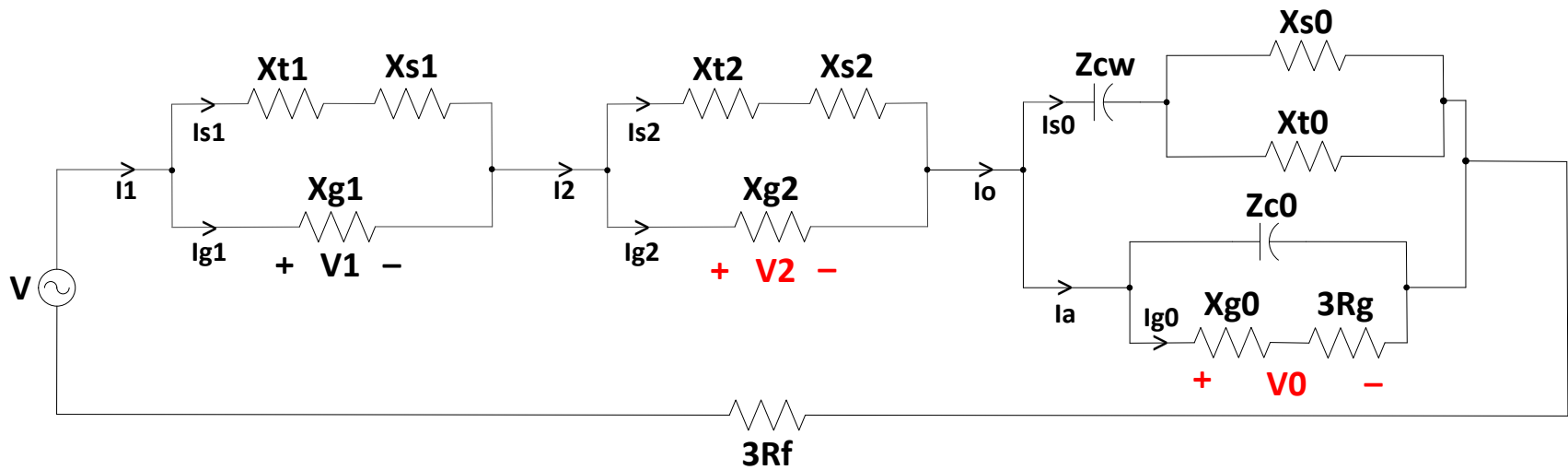
Fault Calculation Examples

Φ G fault on LS of GSU – sequence network connections



Fault Calculation Examples

Φ G fault on LS of GSU – network reduction



Fault Calculation Examples

ΦG fault on LS of GSU

$$Z1 := \frac{(Zs1 + Zt) \cdot Xg1sub}{Zs1 + Zt + Xg1sub} \quad Z2 := \frac{(Zs2 + Zt) \cdot Xg2}{Zs2 + Zt + Xg2} \quad Za := \frac{ZCo \cdot (Xg0 + 3 \cdot Rg)}{ZCo + Xg0 + 3 \cdot Rg} \quad Zb := \frac{Zt \cdot Zs0}{Zt + Zs0} \quad Zc := Zb + ZCw \quad Z0 := \frac{Za \cdot Zc}{Za + Zc}$$

$$Io := \frac{1}{Z1 + Z2 + Z0 + 3 \cdot RfLS} \quad Ip := Io \quad In := Io$$

$$Igl := Ip \cdot \left(\frac{Zs1 + Zt}{Zs1 + Zt + Xg1sub} \right) \quad Is1 := Ip - Igl$$

$$Ig2 := In \cdot \left(\frac{Zs2 + Zt}{Zs2 + Zt + Xg2} \right) \quad Is2 := In - Ig2$$

$$Ia := Io \cdot \left(\frac{Zc}{Za + Zc} \right) \quad Is0 := Io - Ia \quad Ig0 := Ia \cdot \left(\frac{ZCo}{ZCo + Xg0 + 3 \cdot Rg} \right)$$

$$V2 := Ig2 \cdot Xg2$$

$|V2 \cdot 100| = 0.01$ %, and because 0.01% is not > 5%, it will correctly allow a trip for this low side fault

$$V0 := Ig0 \cdot Za$$

$|V0 \cdot 100| = 99$ %, and because 99% is not < 7%, it will correctly allow a trip for this low side fault

References

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- Blackburn, J. L., *Symmetrical Components for Power Systems Engineering*, Marcel Dekker, Inc., copyright 1993
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- Stevenson, W.D., *Elements of Power System Analysis*, McGraw-Hill Book Company, Inc., copyright 1962
- IEEE Standard 242-1986, *IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems*
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